Population Structure and Asset Returns

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Motivation

Baby boomers entering retirement
$\rightarrow$ concerns of diminished returns, compromised pensions

Higher old-age dependency ratio may lead to

- less saving (dissaving) & investment
- shift in asset allocation toward low risk, low return, assets
- reduced labour force growth

With implications for asset returns and retirement outcomes.

Model Framework

Overlapping Generations Model (OLG) with:

- aggregate uncertainty
- two asset classes (risky and risk-free)
- multi-pillar pension systems (saving, pay-go, earnings based)
- endogenous labour supply

→ Generates standard age specific labour, consumption, asset holding, & portfolio allocation qualitatively consistent with data

→ Older population → moderately lower asset returns

Demographics

- Overlapping generations, $j \in \{1,2,\dots,20\}$, ages 18 – 97
- Five life stages: YW, MW, W, SR, R
- Intra-cohort heterogeneity, $i \in \{1,2\}$, baseline $i = 1$
- fertility rate: n
- survival probability: $\phi_j^i \in \{1,2\}$, $\phi_J^i = 0$

$$N^{\mathrm{i}}_{\mathrm{j,\,t}} = \begin{cases} (1+n)\chi^i N_{\mathrm{0,t\text{-}1}}, & \text{if } j=1, \\ \phi^i_{j\text{-}1}\chi^i N_{\mathrm{j\text{-}1,\,t\text{-}1}}, & \text{if } 1<j\leq J. \end{cases}$$

Household Time Endowment

$$H_j = \begin{cases} H\left(1 - FC_j - FE_j\right), & \text{if } j \in \{YW, MW\}, \\ H, & \text{if } j \in \{W, SR, R\}. \end{cases} \qquad (2.1)$$

- Fixed constant H units of time
- Education (FE) and child rearing (FC)
- SR can work maximum of $\iota_p H$

Household Preferences

Periodic utility from Consumption and Leisure

$$u^i(c,h) = \frac{c^{1-\gamma_c}}{1-\gamma_c} + \Psi \frac{(H_j - h)^{1-\gamma_h}}{1-\gamma_h}$$

- Coefficient of relative risk aversion: γ_c
- Parameter that regulates Frisch elasticity of labour supply: γ_h
- Utility weight of leisure relative to consumption: Ψ

Assets

Total Asset Holdings: $\theta^{i}_{j,t}$

Risk Free Bonds

- Return in period t+1: $\bar{r}_t$
- Share of total assets in risk free: $\eta^{i}_{j,t}$
- Zero net supply: $\sum_j \sum_i \eta^{i}_{j,t}\, \theta^{i}_{j,t}\, N^{\mathrm{i}}_{\mathrm{j,t}} = 0$ (2.2)

Risky Capital

- Return in period t+1: r_{t+1}
- Share of total assets: $1 - \eta^{i}_{j,t}$
- Total capital: $K_t = \sum_j \sum_i \left(1 - \eta^{i}_{j,t-1}\right) \theta^{i}_{j,t-1}\, N^{\mathrm{i}}_{\mathrm{j,t-1}}$ (2.3)

Production

$$Y_t = z_t K_t^{\alpha} H_t^{1-\alpha} \quad \text{and} \quad K_{t+1} = (1-\delta) K_t + q_t I_t$$

$$\ln(z_t) = \rho \ln(z_{t-1}) + \nu_t \quad \text{where} \quad \nu_t \sim N(0, \sigma_z^2)$$

$$\ln(q_t) = \rho_q \ln(q_{t-1}) + \nu_{q,t} \quad \text{where} \quad \nu_{q,t} \sim N(0, \sigma_q^2)$$

- Aggregate efficient labour is: $H_t = \sum_j \sum_i \varepsilon_j^i h_{j,t}^i N_{j,t}^i$ (2.4)
- Baseline: $\varepsilon_j^i = 1$ → no age & type-specific labour productivity.
- $corr(\sigma_q^2, \sigma_z^2) = 0$

Pay-as-you-go Pension

Pay-as-you-go proportional pension scheme

$$p_{j,t} = \begin{cases} 0, & \text{if } j \in \{YW, MW, W\}, \\ \frac{\tau_s w_t H_t}{\sum_{j \in \{OW, R\}} \sum_i N^i_{j,t}} & \text{if } j \in \{SR, R\}. \end{cases} \quad (2.5)$$

- Fixed tax, τ_s , on labour income uniformly distributed to retirees.

Partially Funded Pension

Partially funded, employment earnings based pension

$$p^G_{\mathrm{j,\,t}} = \begin{cases} 0, & if\ j \in \{YW, MW, W\}, \\ \kappa_{\mathrm{j}} \left(\dfrac{w_{\mathrm{ss}} \sum_i \varepsilon^i_{SR\text{-}1} h^{\mathrm{i}}_{\mathrm{SR\text{-}1,\,SS}} N^{\mathrm{i}}_{\mathrm{SR\text{-}1,\,SS}}}{\sum_i N^{\mathrm{i}}_{\mathrm{SR\text{-}1,\,SS}}} \right) & if\ j \in \{SR, R\}. \end{cases} \qquad (2.6)$$

- Government taxes working cohorts at rate τ^G_s , and pays out fraction κ_{j} of pre-retirement income.

Government Budget

In the three pillar model:

$$\sum_{\mathrm{j=SR}}^{R} p_j^G N_{\mathrm{j,t}}^i = [\eta_G(1+(1-\tau_r)\bar{r}_{\mathrm{t-1}})+(1-\eta_G)(1+(1-\tau_r)r_t)]\theta_G+\tau_{\mathrm{s}}^{\mathrm{G}} w_t H_t + B_t^G \qquad (2.7)$$

Aggregate Asset holdings in the three pillar model:

$$\sum_{\mathrm{j}} \sum_i \eta_{j,t}^i \theta_{j,t}^i N_{\mathrm{j,t}}^i + \eta_G \theta_G = B_t^G$$

$$K_t = \sum_{\mathrm{j}} \sum_i (1-\eta_{j,t-1}^i)\theta_{j,t-1}^i N_{\mathrm{j,t-1}}^i + (1-\eta_G)\theta_G$$

- Government holds pool of assets, θ_G, with proportion η_G in risk-free bonds, and issues bonds B_t^G to balance budget.

Taxes and Bequests

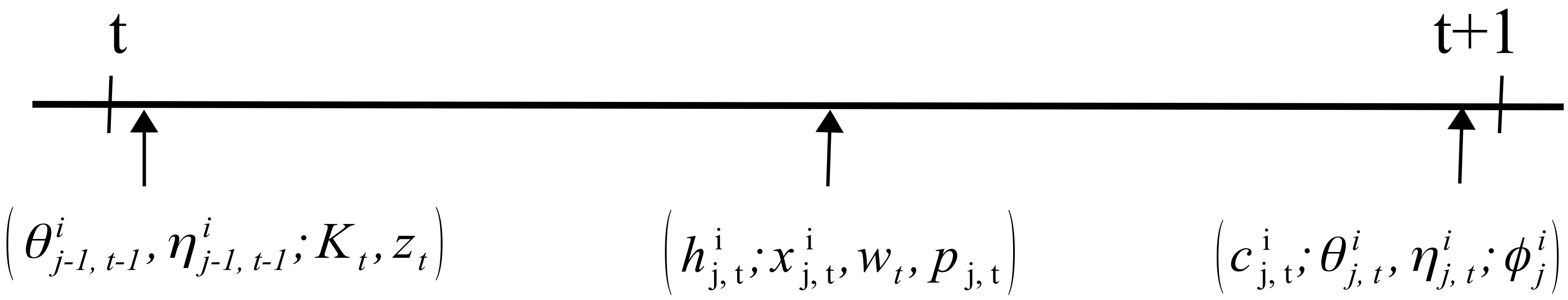
Taxes

- Consumption tax: τ_c
- Labour Income tax: τ_h
- Investment income tax: τ_r
- Tax on pension income: τ_p
- Tax for pay-go pension and social security: τ_s and τ_s^G

Bequests

- Base model has accidental bequests only.
- Bequest motive – utility from leaving bequest $\nu(X) = \Gamma \frac{X^{1-\gamma_b}}{1-\gamma_b}$

Timeline and State Space (s_t ; z_t)



$s_t = \left(x^1_{2,t}, \ldots, x^i_{j,t}, \ldots, x^I_{J,t}; z_t\right)$, where $x^i_{j,t}$ is the value of asset holdings pd t

$$x^i_{j,t} = \left[\eta^i_{j-1,\, t-1}\left(1+\left(1-\tau_r\right)\bar{r}_{t-1}\right) + \left(1-\eta^i_{j-1,\, t-1}\right)\left(1+\left(1-\tau_r\right)r_t\right)\right]\theta^i_{j-1,\, t-1}$$

Household Decision

$$V^{i}_{j}(s_t; z_t) = \max_{\{c^{i}_{j,t}, h^{i}_{j,t}, \theta^{i}_{j,t}, \eta^{i}_{j,t}\}} u^{i}(c^{i}_{j,t}, h^{i}_{j,t}) + \beta \phi^{i}_{j} E_t\left[V^{i}_{j+1}(s_{t+1}; z_{t+1})\right]$$

s.t.

$$(1+\tau_c)c^{i}_{j,t} + \theta^{i}_{j,t} \leq \left\{(1-\tau_s-\tau^{G}_{s}-\tau_h)w_t \varepsilon^{i}_{j} h^{i}_{j,t} + x^{i}_{j,t} + (1-\tau_p)(p_{j,t} + p^{G}_{j}) + \xi_t - HC\right\}$$

where

$$h^{i}_{j,t} \leq H^{c}_{j} = \begin{cases} H_j, & \text{if } j \in \{YW, MW, W\}, \\ \iota_p H, & \text{if } j \in \{SR\}, \\ 0, & \text{if } j \in \{R\}, \end{cases} \quad \&$$

$$HC_j = \begin{cases} 0, & \text{if } j \in \{YW, MW, W\}, \\ 0.2 \exp\left(\frac{4(j-12)}{J-12} - 4\right), & \text{if } j \in \{SR, R\}. \end{cases}$$

Household Decision – oldest generation

$$V_J^i(s_t; z_t) = \max_{\{c_{J,t}^i, \theta_{J,t}^i, \eta_{J,t}^i\}} u^i(c_{J,t}^i, 0) + \beta E_t\left[\nu^i(X_{J+1,t+1}^i)\right]$$

where

$$X_{J+1,t+1}^i = \left[\eta_{J,t}^i(1+(1-\tau_r)\bar{r}_t) + (1-\eta_{J,t}^i)(1+(1-\tau_r)r_{t+1})\right]\theta_{J,t}^i$$

and

$$\nu(X) = \Gamma \frac{X^{1-\gamma_b}}{1-\gamma_b}$$

Solution to Household Problem

For j<J

$$\left(c^{\mathrm{i}}_{\mathrm{j,t}}\right)^{-\gamma_c} = \beta\phi^i_j E_t\left[\left(1+\left(1-\tau_r\right)r_{\mathrm{t+1}}\right)\left(c^{\mathrm{i}}_{\mathrm{j+1,t+1}}\right)^{-\gamma_c}\right], \tag{3.11}$$

$$0 = \beta\phi^i_j E_t\left[\left(1-\tau_r\right)\left(\bar{r}_t - r_{\mathrm{t+1}}\right)\left(c^{\mathrm{i}}_{\mathrm{j+1,t+1}}\right)^{-\gamma_c}\right], \tag{3.12}$$

$$\frac{\psi^i\left(H_j - h^{\mathrm{i}}_{\mathrm{j,t}}\right)^{-\gamma_h} + \lambda^2_{j,t}}{\left(c^{\mathrm{i}}_{\mathrm{j,t}}\right)^{-\gamma_c}} = \frac{1-\tau_s-\tau^G_s-\tau_h}{1+\tau_c}\; w_t\varepsilon^i_j\ , \tag{3.13}$$

$$\lambda^2_{j,t}\left(H^{\mathrm{c}}_{\mathrm{j}} - h^{\mathrm{i}}_{\mathrm{j,t}}\right) = 0 \tag{3.14}$$

For j=J

$$\left(c^{\mathrm{i}}_{\mathrm{J,t}}\right)^{-\gamma_b} = \beta\Gamma E_t\left[\left(1+\left(1-\tau_r\right)r_{\mathrm{t+1}}\right)\left(X^{\mathrm{i}}_{\mathrm{J+1,t+1}}\right)^{-\gamma_b}\right],$$

$$0 = \beta\Gamma E_t\left[\left(1-\tau_r\right)\left(\bar{r}_t - r_{\mathrm{t+1}}\right)\left(X^{\mathrm{i}}_{\mathrm{J+1,t+1}}\right)^{-\gamma_b}\right]$$

Firm Decision

Firm maximizes profits, resulting in:

$$r_t = \alpha z_t K_t^{\alpha-1} H_t^{1-\alpha} - \delta \ , \tag{3.15}$$

$$w_t = (1-\alpha) z_t K_t^{\alpha} H_t^{-\alpha}. \tag{3.16}$$

where $\delta \in [0,1]$.

Recursive Competitive Equilibrium

- Value functions $V_{\mathrm{j}}^{\mathrm{i}}(s_t;z_t)$,
- Household policy functions for consumption, $c_{\mathrm{j,t}}^{\mathrm{i}}(s_t;z_t)$, labour supply, $h_{\mathrm{j,t}}^{\mathrm{i}}(s_t;z_t)$, total saving, $\theta_{j,t}^{i}(s_t;z_t)$, and share of saving invested in risk-free bonds, $\eta_{j,t}^{i}(s_t;z_t)$,
- Inputs for the representative firm $K_{\mathrm{t}}(s_t;z_t)$ and $H_{\mathrm{t}}(s_t;z_t)$,
- Government policy, $p_{\mathrm{t}}(s_t;z_t)$ and $B_{\mathrm{t}}^{G}(s_t;z_t)$,
- Rates of return $\bar{r}_{\mathrm{t}}(s_t;z_t)$ and $r_{\mathrm{t}}(s_t;z_t)$, and wage $w_{\mathrm{t}}(s_t;z_t)$,

Such that in each period the:

- household problems are solved,
- the competitive firm maximizes profits,
- all markets clear.

Parameterization

Base model, with J =20, i =1, χ =1, ε =1, HC=0, Γ=0, and sets several parameters fixed and exogenous to the model:

Parameter	Value	Description
H	4	Time available to household (one period represents 4 yrs)
β	0.8515	Discount factor (0.95 annual)
α	0.3	Capital's share of production
ρ_z	0.4401	Autocorrelation coefficient for TFP
σ_z	0.0305	Std. Deviation of error for TFP process
ρ_q	0.4401	Autocorrelation coefficient for IST
σ_q	0.1221	Std. Deviation of error for IST process
δ	0.192	Depreciation Rate
n	0.0489	Population Growth rate
γ_c	2.0	Relative risk aversion – consumption
γ_b	2.0	Relative risk aversion - bequest
γ_l	3.0	Inverse of intertemporal elasticity of substitution of non-market time
Ψ	21.833	Utility weight of non-market time relative to consumption
τ_c, τ_r, τ_p	0.123, 0.167, 0.167	Tax rates on consumption, investment income, pension,
$\tau_h + \tau_s + \tau_s^G$	0.167	Tax on labour income
$ratio_s$	1.0	Proportion of labour tax to social security
ι_p	0.08	Labour constraint for SR

Lifecyle Consumption, Labour, & Asset Profiles

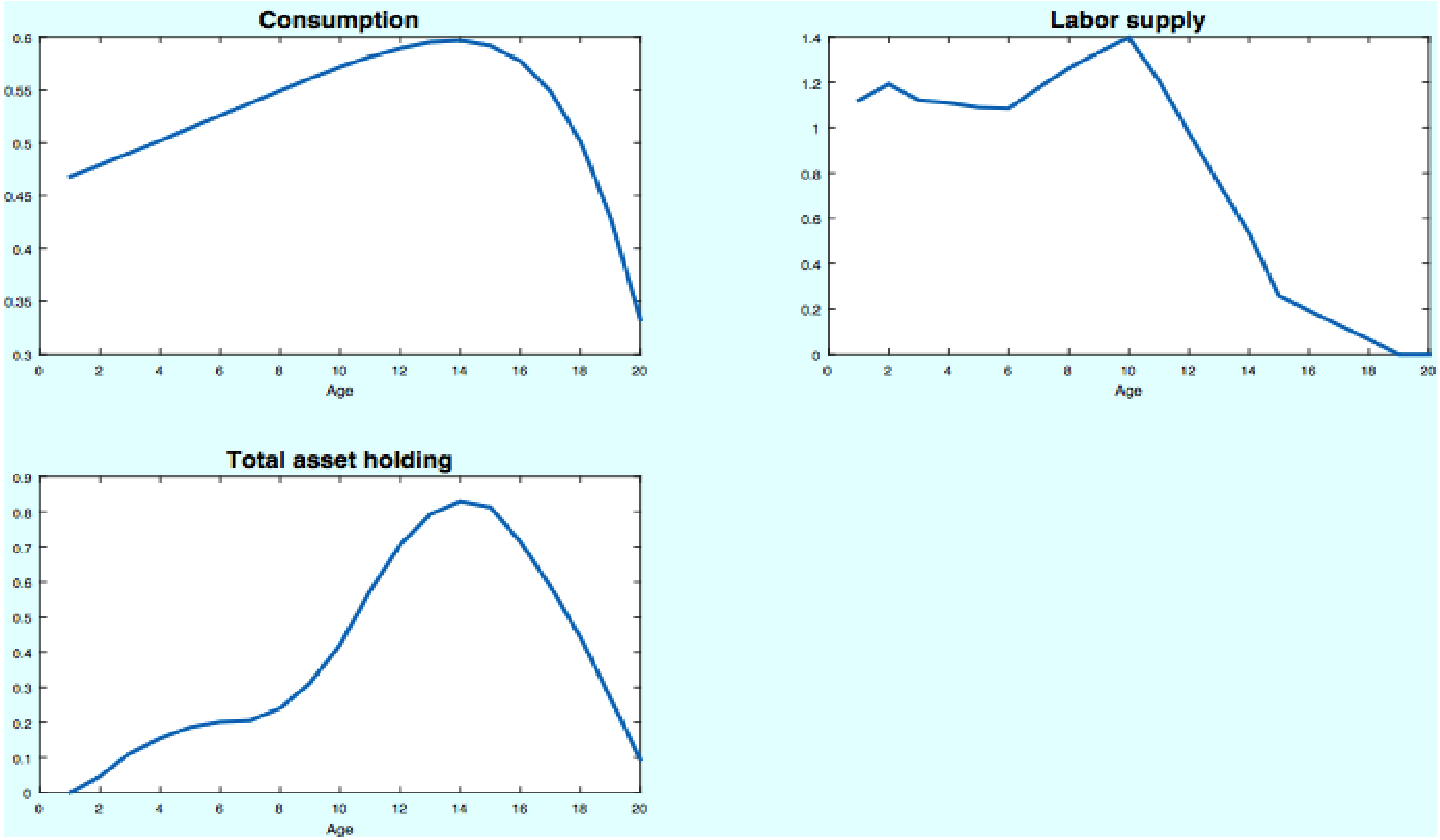


Figure 1 – Lifecycle consumption, labour and asset profiles

Observed Age-Specific Portfolio Allocation

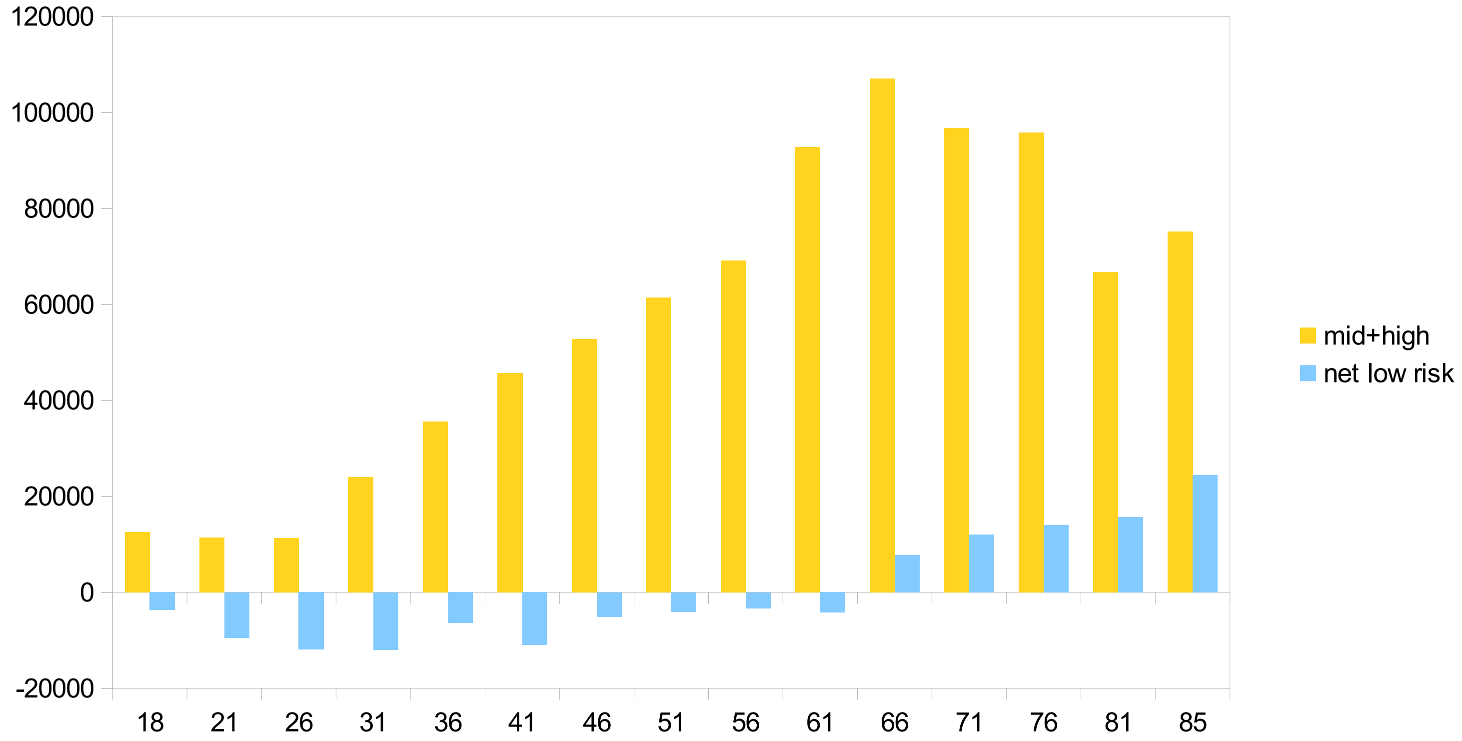


Figure 2 –Portfolio allocation by age: risky vs net low-risk financial assets

Portfolio Allocation – 2 pillar pension model

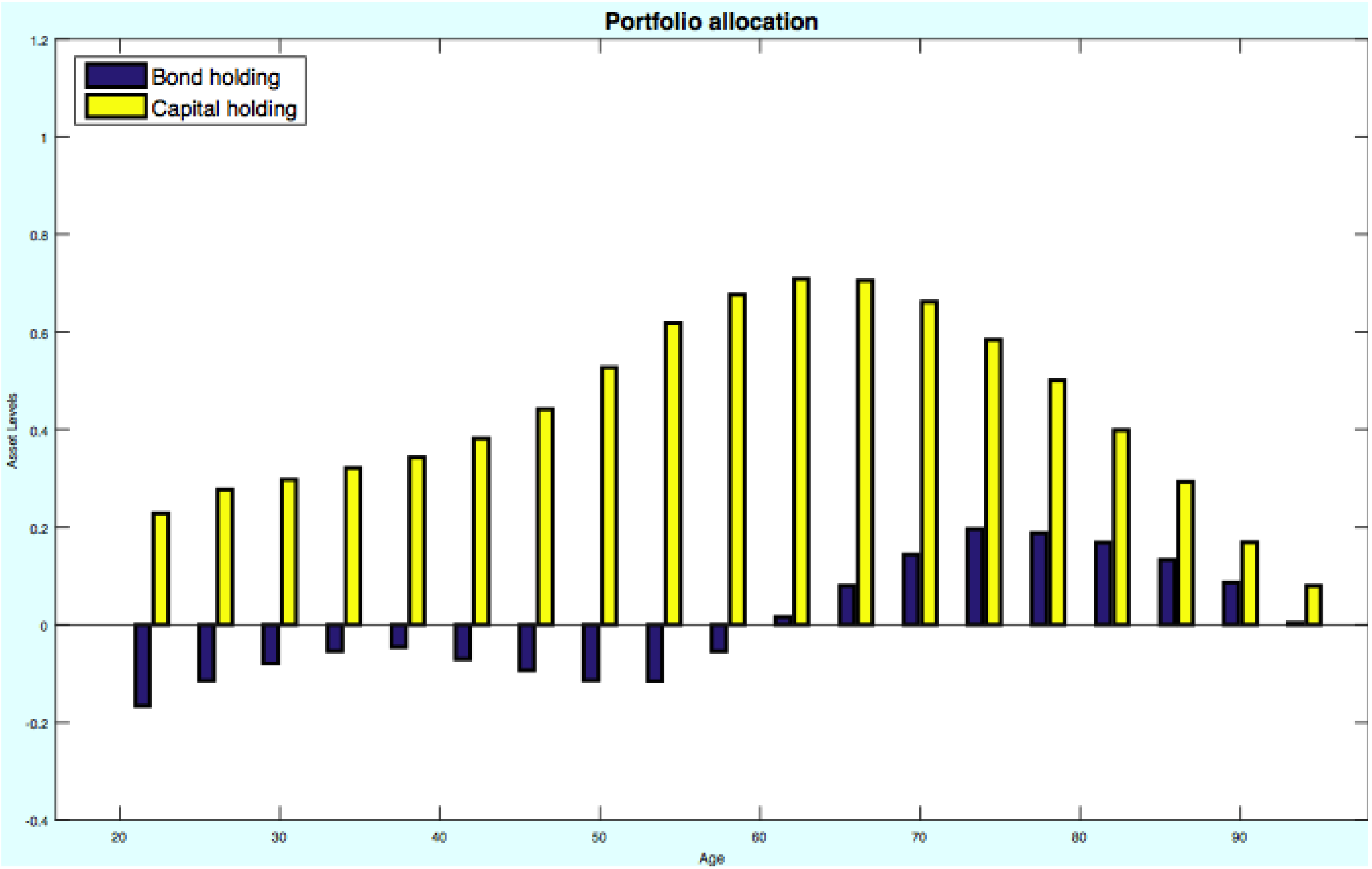


Figure 3 – Age-specific portfolio allocation in 2 pillar model

Portfolio Allocation – 3 pillar pension model – baseline

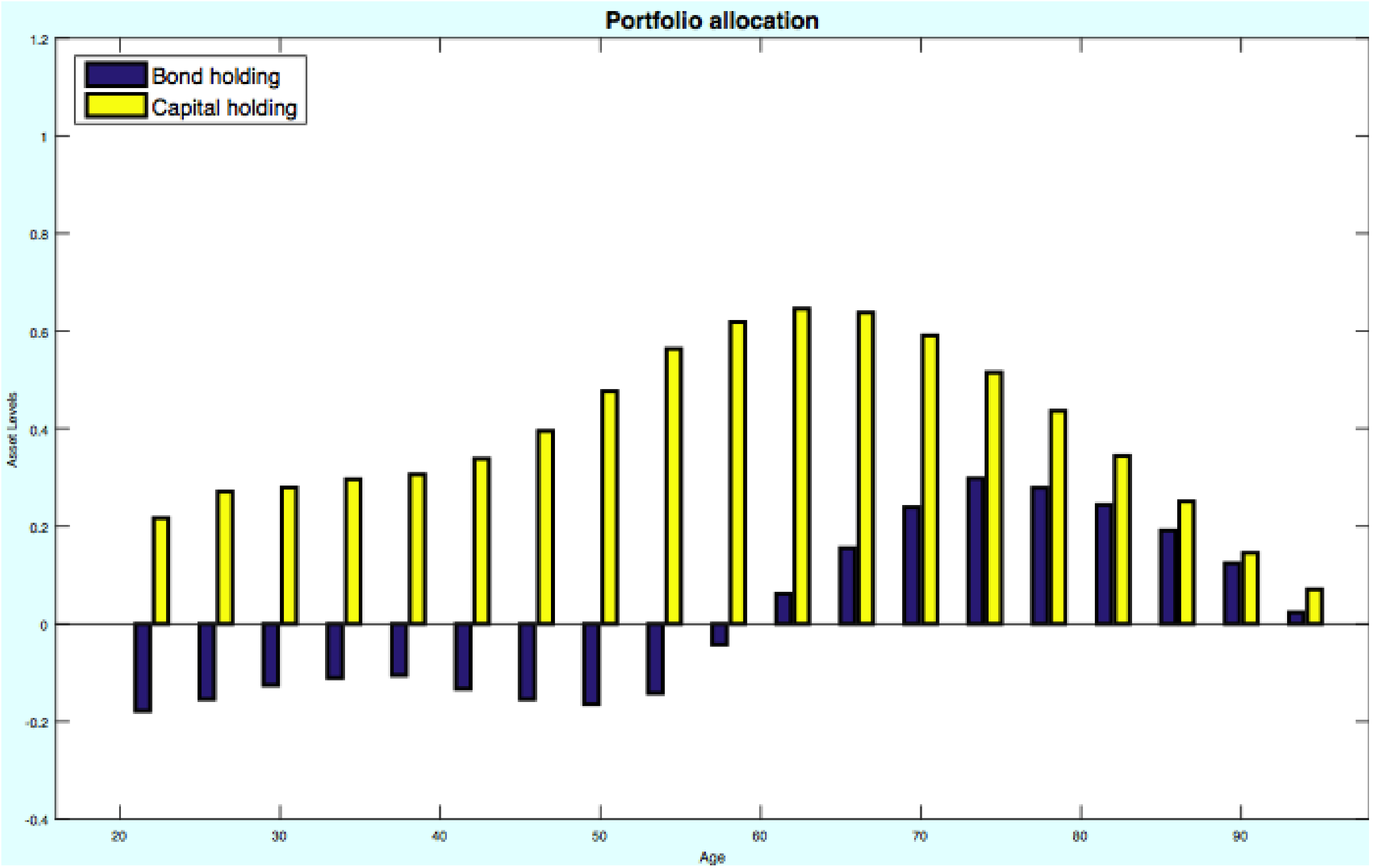


Figure 4 – Age-specific portfolio allocation in 3 pillar model

3-pillar Model Results under Alt. Demog. Structures

Variable	**Base-3pillar**	**+10%**	**+20%**	**-10%**	**-20%**
$E_t(r_{t+1})$	0.2855	0.2788	0.2735	0.2919	0.2965
$\bar{r}_t$	0.2851	0.2784	0.2730	0.2915	0.2961
Prv risky assets/GDP	0.5223	0.5233	0.5362	0.5214	0.5206
$c_{20,t}$	0.3327	0.3771	0.4183	0.2984	0.2512

- Model predicts modest differences.

Portfolio Allocation - Alternative Replacement Ratio

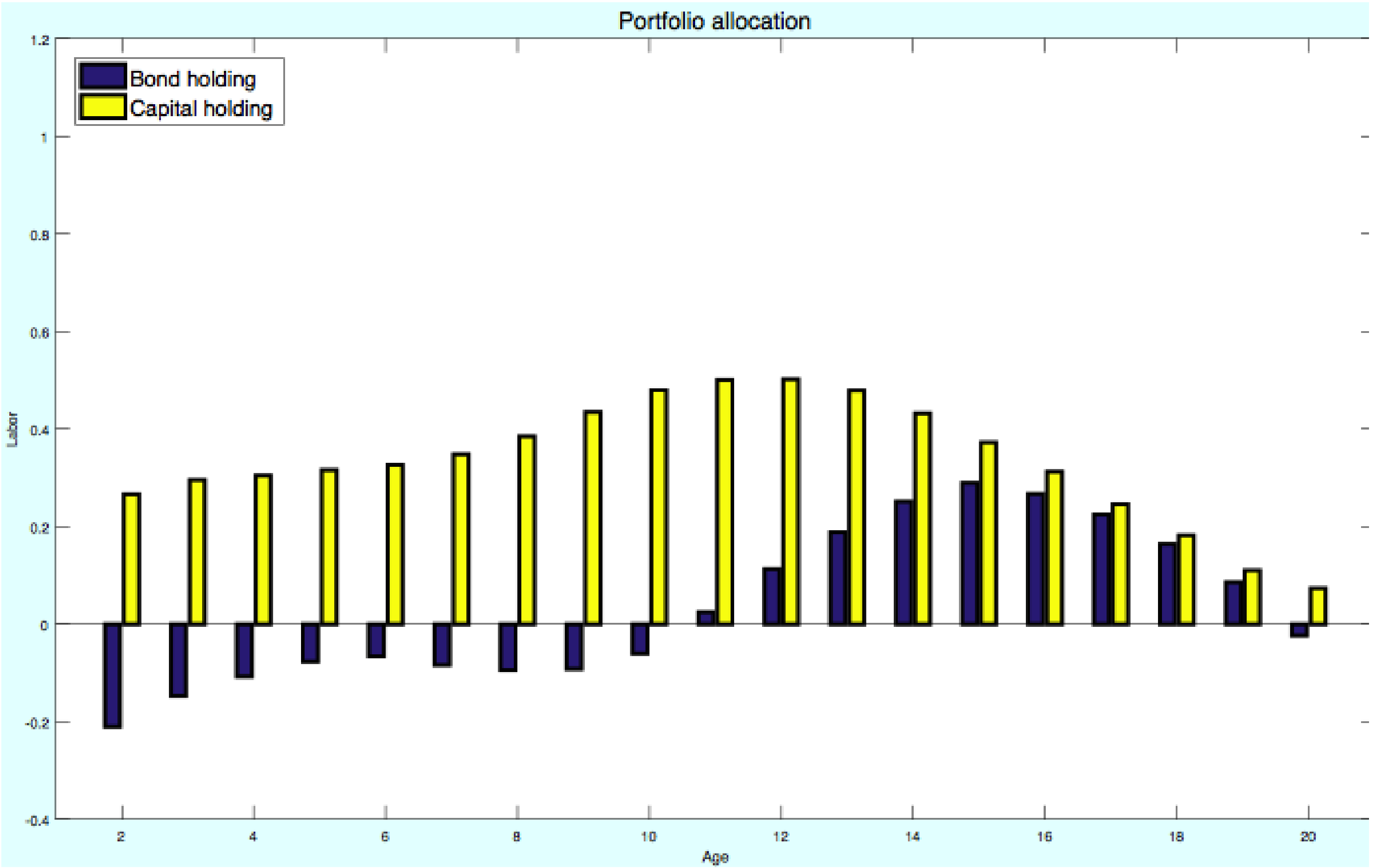


Figure 5– Age-specific portfolio allocation, high replacement ratio, κ =0.4

Portfolio Allocation– 3 pillar + health costs + bequest

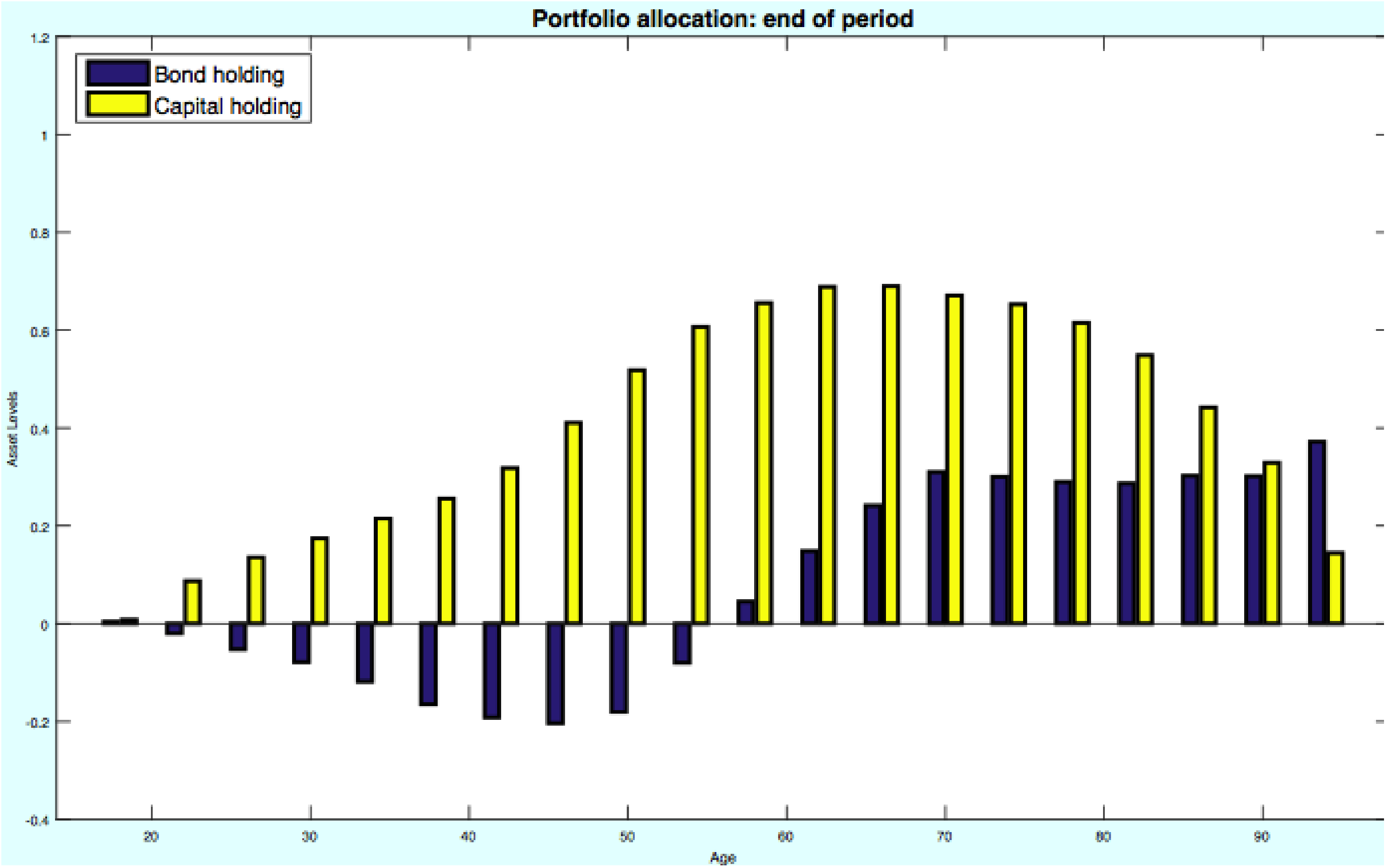
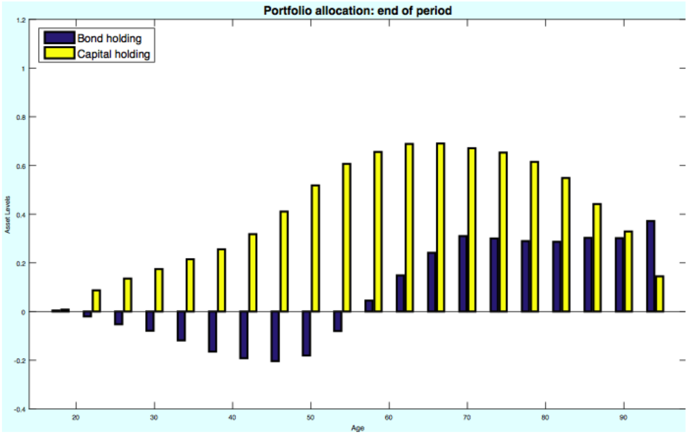
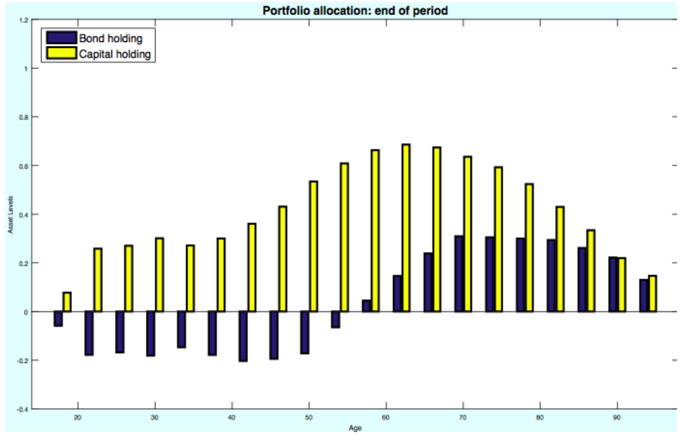


Figure 6 – Age-specific portfolio allocation, 3 pillar +bequest +health cost

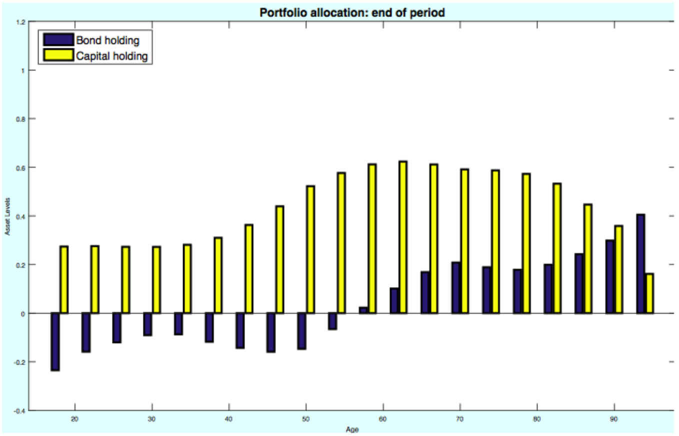
Portfolio Allocation under alternative models



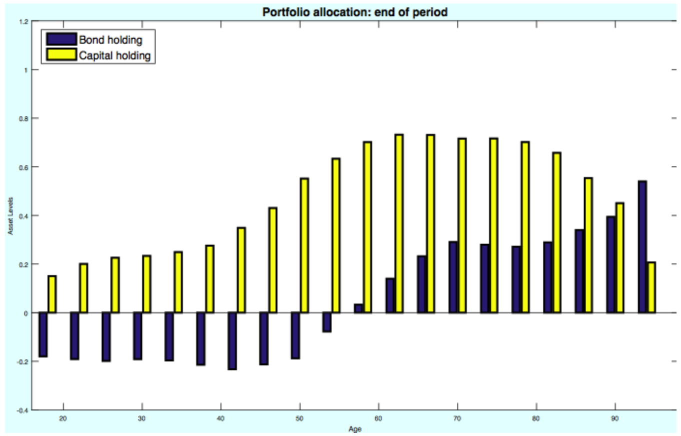
(a) Baseline + Bequest (Γ=4, γ_b=2=γ_c) + Health Cost



(b) Baseline + Bequest (as a luxury good) + Health Costs



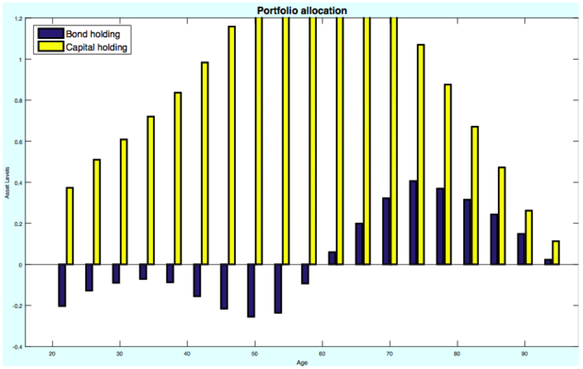
(c) Baseline + Bequest + Health Cost+ Work in R (SR= .12, R= .8)



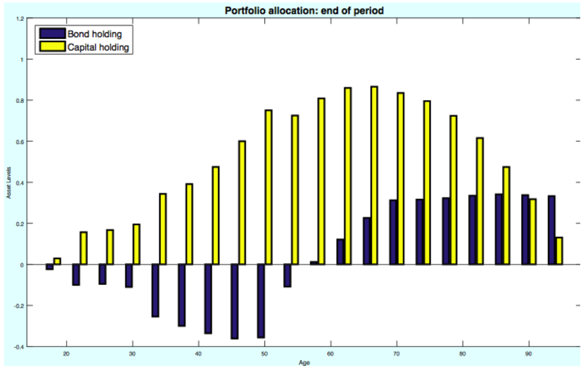
(d) Base + Beq + Health Cost+ Work in R + Quadratic productivity

Figure 7 – Age-specific portfolio allocation, alternative models

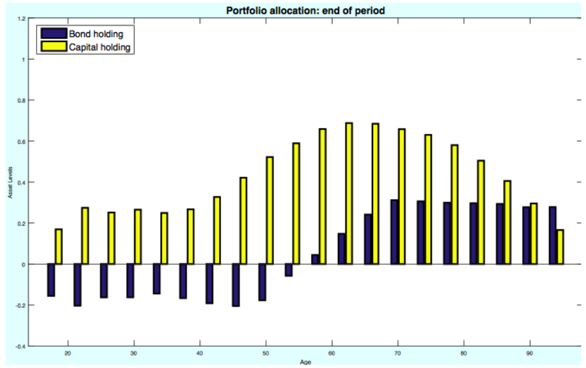
Sensitivity Analysis



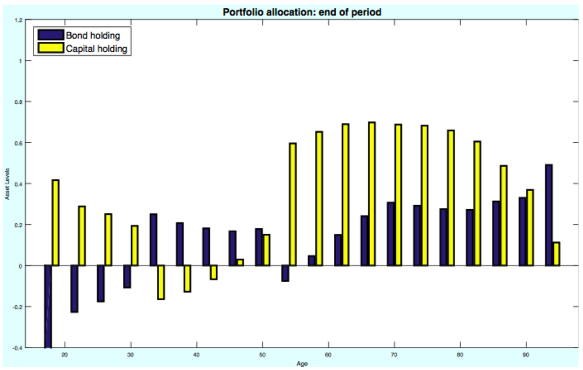
(a) Baseline with higher discount factor, β (0.999)



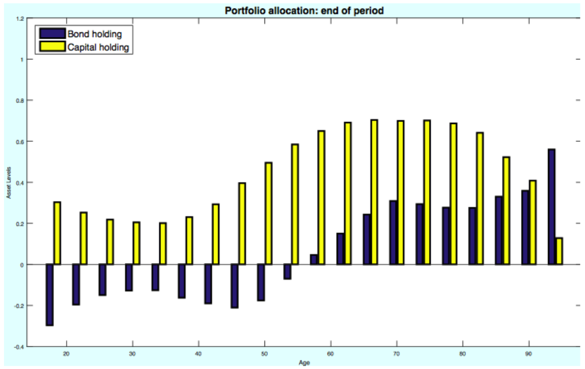
(b) Base + Beq + Health Cost, higher risk avers (γ_c=3)



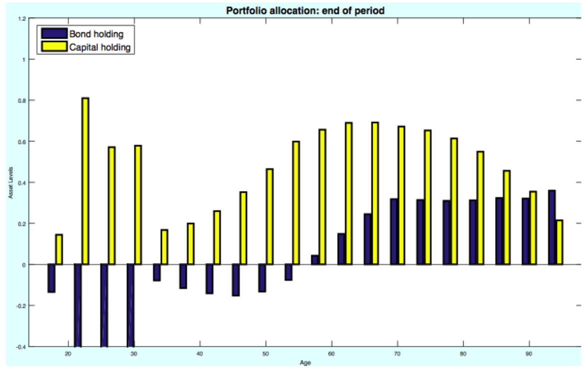
(c) Base + Beq(Γ=4, γ_b=1.5)+ Health Cost
low curv on bequest



(d) Base + Bequest (Γ=4,γ_b=3) + Health Cost
high curv on bequest



(e) Baseline + Bequest (Γ=6, γ_b=3) + Health Cost



(f) Base + Beq (Γ=6, γ_b=1.5) + Health Cost

Figure 8 – Age-specific portfolio allocation, alternative parameter values

Discussion and Next Steps

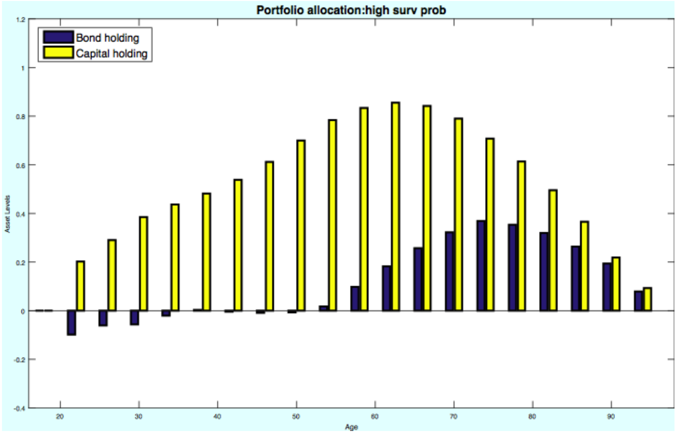
- Asset prices are moderately lower with older population:
  Higher survival probability for age 65+ (max20% at j=J)
  → approximately 4% lower returns on capital and on bonds
- Higher replacement ratio → lower asset accumulation

Next steps:

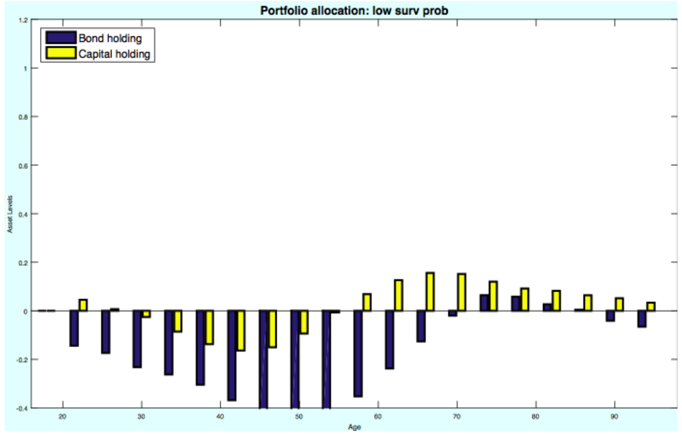
- Improve portfolio allocation match
  → consumption saturation
  → intra-cohort heterogeneity
- Explore further intra-cohort heterogeneity models

Appendix

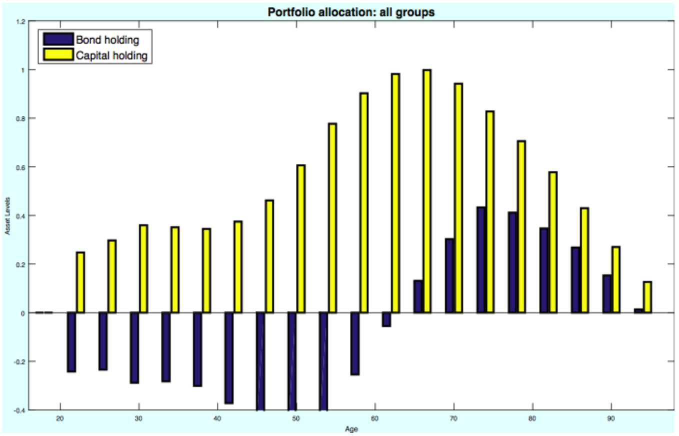
Heterogeneity – high and low survival rate



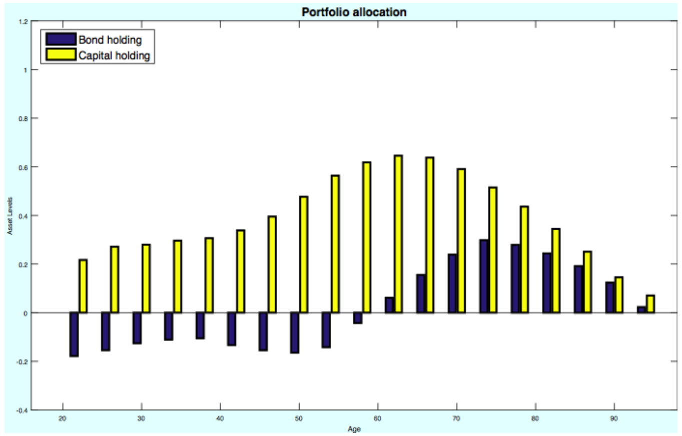
(a) 3-Pillar pension with intra-cohort heterogeneity, high survival



(b) 3-Pillar pension with intra-cohort heterogeneity, low survival



(c) 3-Pillar pension wit intra-cohort heterogeneity, aggregate



(d) Baseline, no intra-cohort heterogeneity

Figure 9– Age-specific portfolio allocation with intra-cohort heterogeneity

Heterogeneity – high and low survival rate (cont)

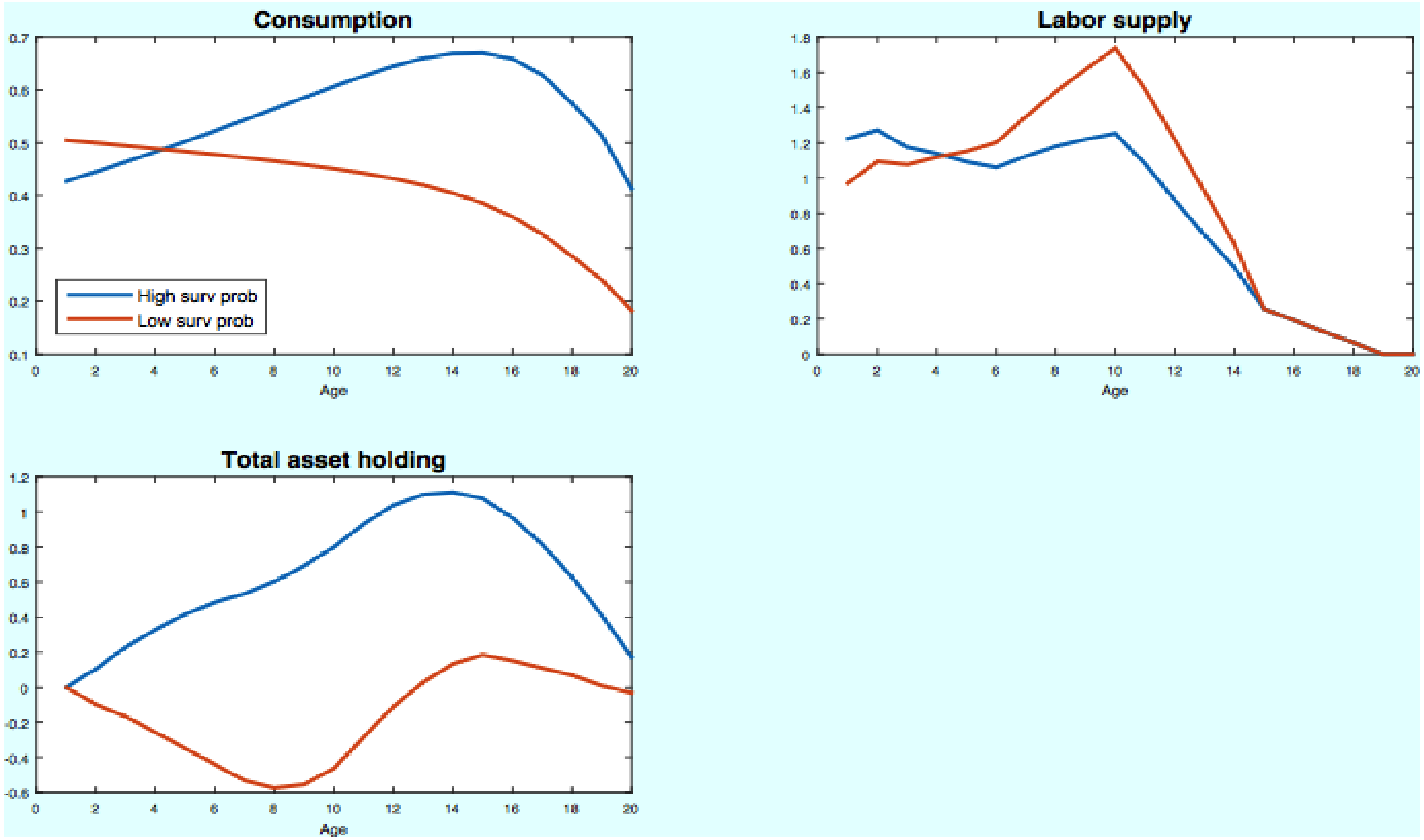
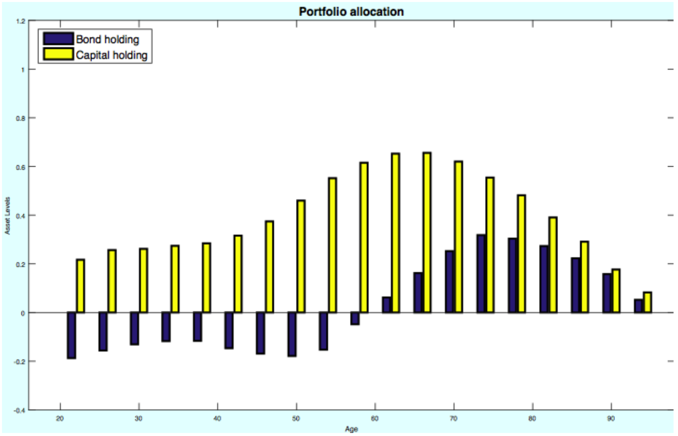
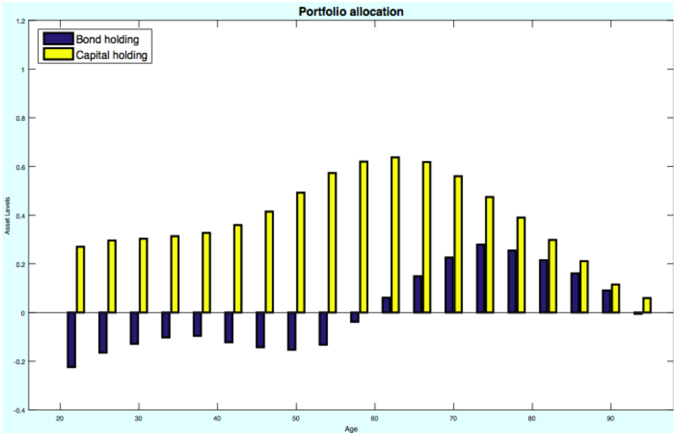


Figure 10 – Consumption, labour & asset profiles under heterogeneity

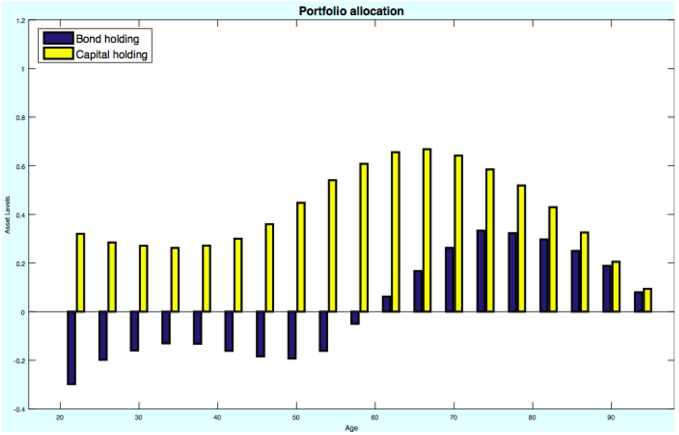
Portfolio allocation under Alt. Demog. Structures



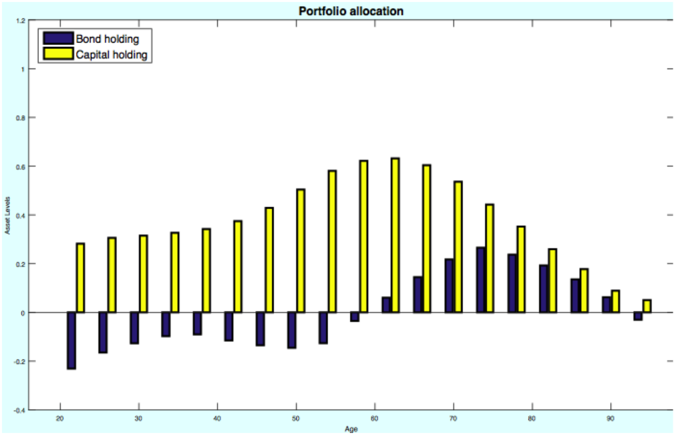
(a) Baseline + 10% maximal higher survival probability



(b) Baseline - 10% maximal higher survival probability



(c) Baseline + 20% maximal higher survival probability



(d) Baseline - 20% maximal lower survival probability

Figure 11 – Age-specific portfolio allocation, alternative demographics