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Role of S_κ in the CMI Mortality Projection Model

Steven Rimmer

Actuarial Teachers and Researchers Conference, 18 July 2017

Agenda

- Warm-up: Lee-Carter model
- Overview of the CMI Mortality Projection Model
- Impact of varying the period smoothing parameter
- Application on other data sets

- *Member of the CMI committee.*
  Comments presented today are own thoughts made in a personal capacity.

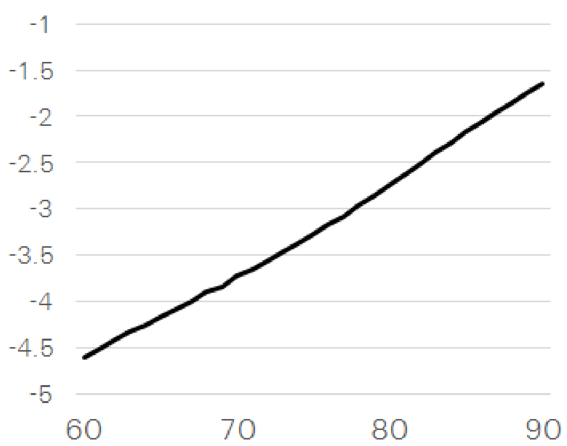
Warm-up: Lee-Carter mortality model

Assume: $\log(m_{xt}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{xt}$

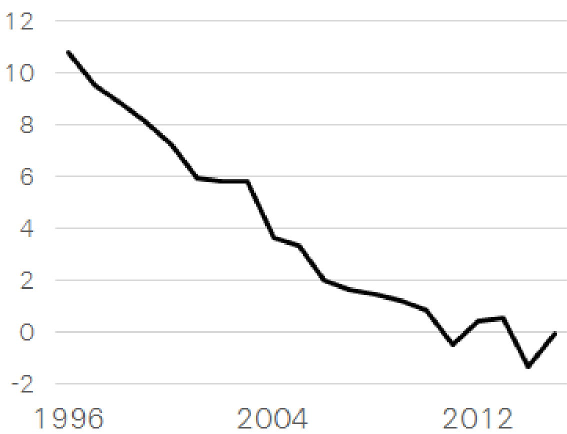
Find: α_x , β_x , and κ_t

Minimise: $\text{Deviance} = 2\sum_{x,t}\left(D_{x,t}\log D_{x,t} - D_{x,t} - D_{x,t}\log E_{x,t}m_{x,t} + E_{x,t}m_{x,t}\right)$

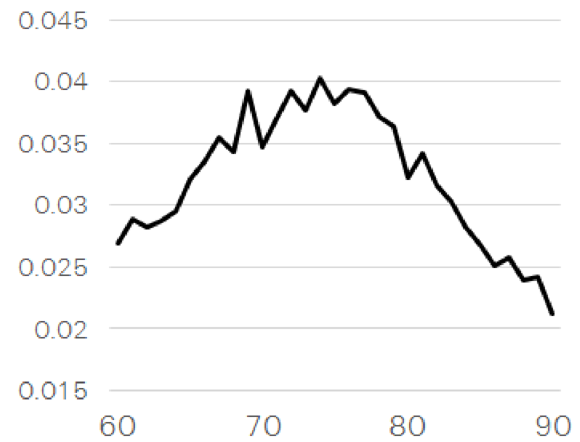
α_x



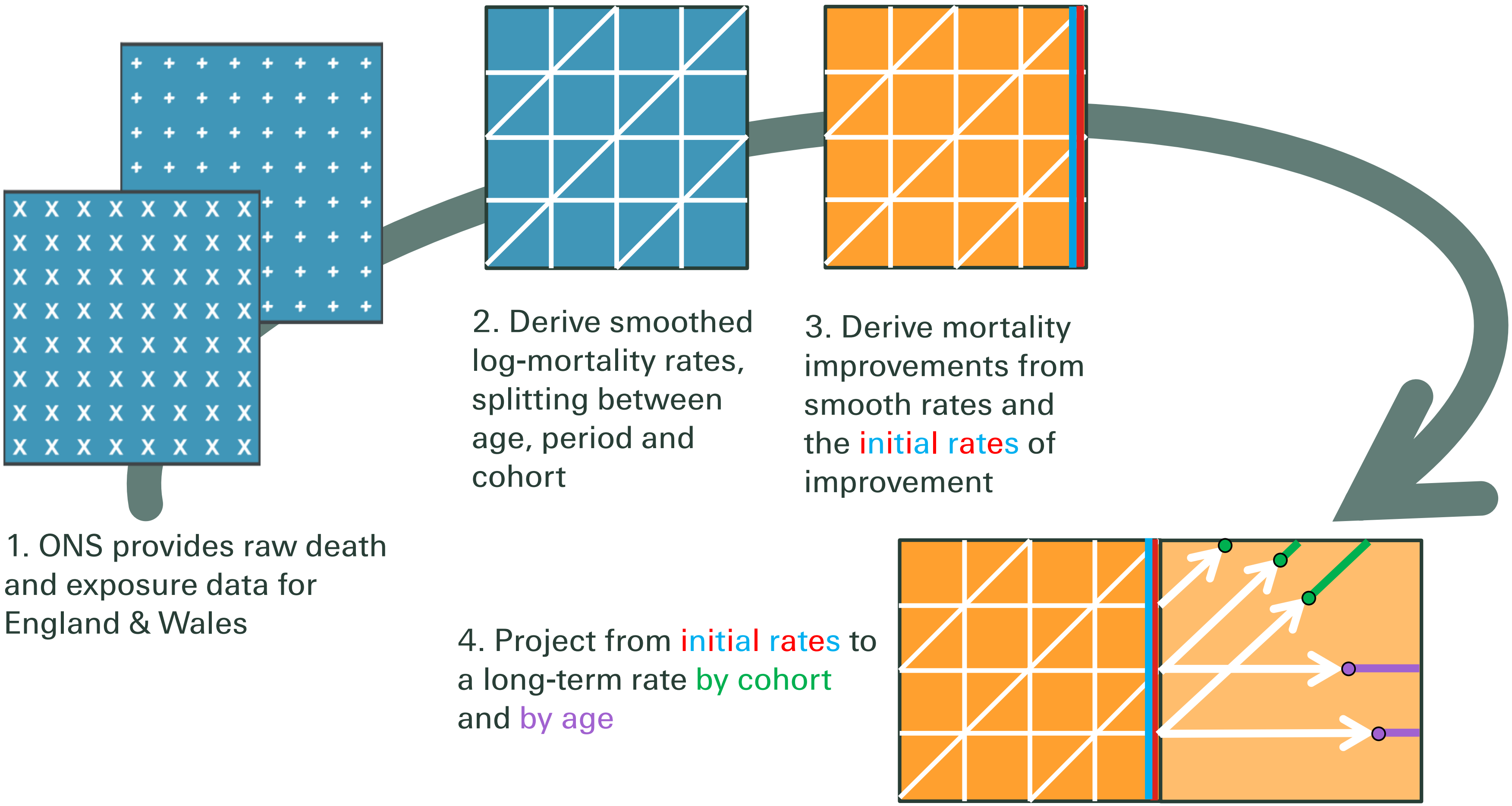
κ_t



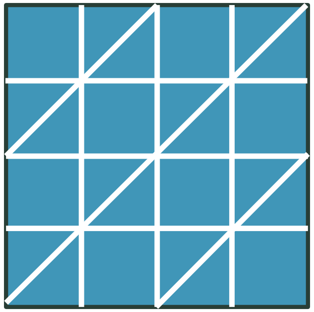
β_x



CMI Projection Model overview: Smoothed decomposition into age-period and cohort terms, separately projected to long-term rate(s)



Unsmoothed, decomposed fit: log-mortality rates

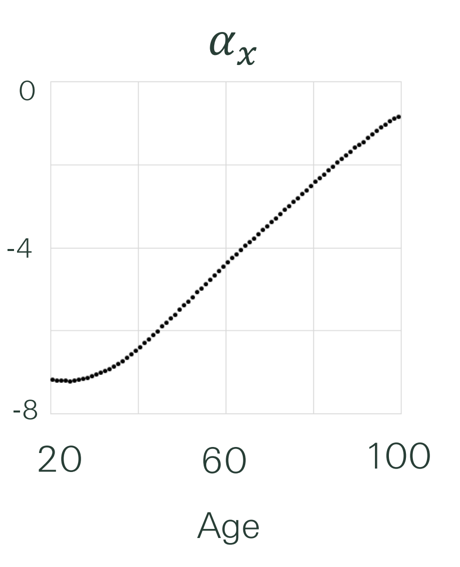


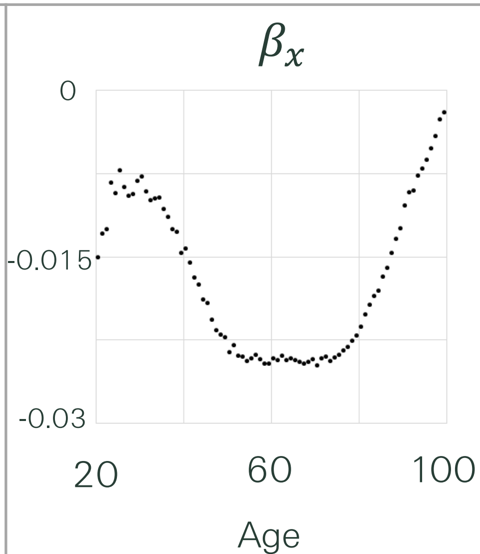
Define: $\log m_{x,t} = \alpha_x + \beta_x(t - \bar{t}) + \kappa_t + \gamma_{t-x}$

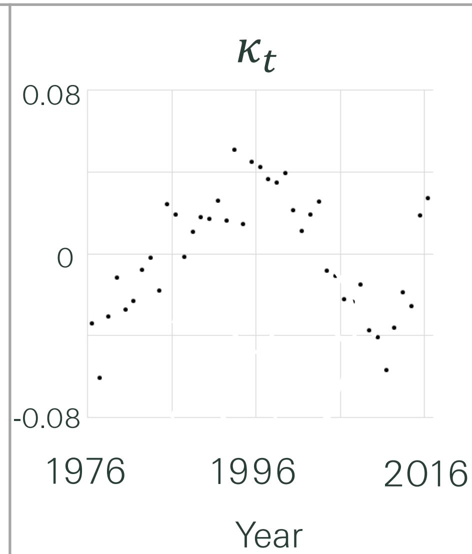
Find: $\alpha_x, \beta_x, \kappa_t, \gamma_{t-x}$

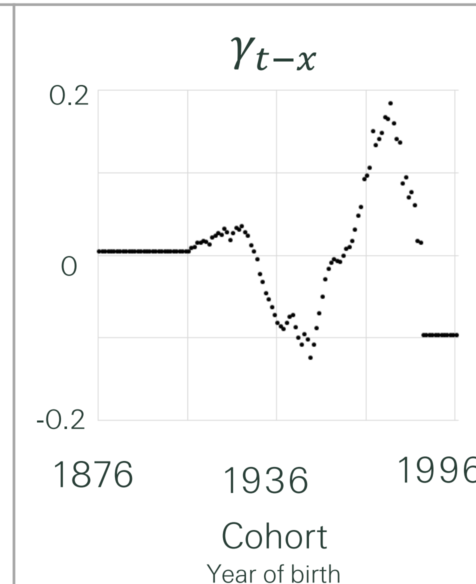
To minimise: $\text{Deviance} = 2\sum_{x,t}\left(D_{x,t}\log D_{x,t} - D_{x,t} - D_{x,t}\log E_{x,t}m_{x,t} + E_{x,t}m_{x,t}\right)$

(Subject to identifiability constraints)

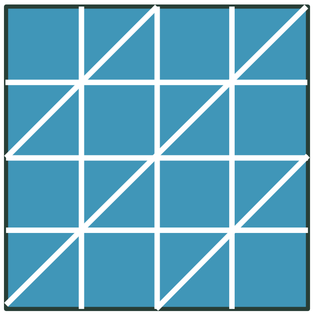








Smoothed, decomposed fit: log-mortality rates



Define:

$$\log m_{x,t} = \alpha_x + \beta_x(t - \bar{t}) + \kappa_t \; + \gamma_{t-x}$$

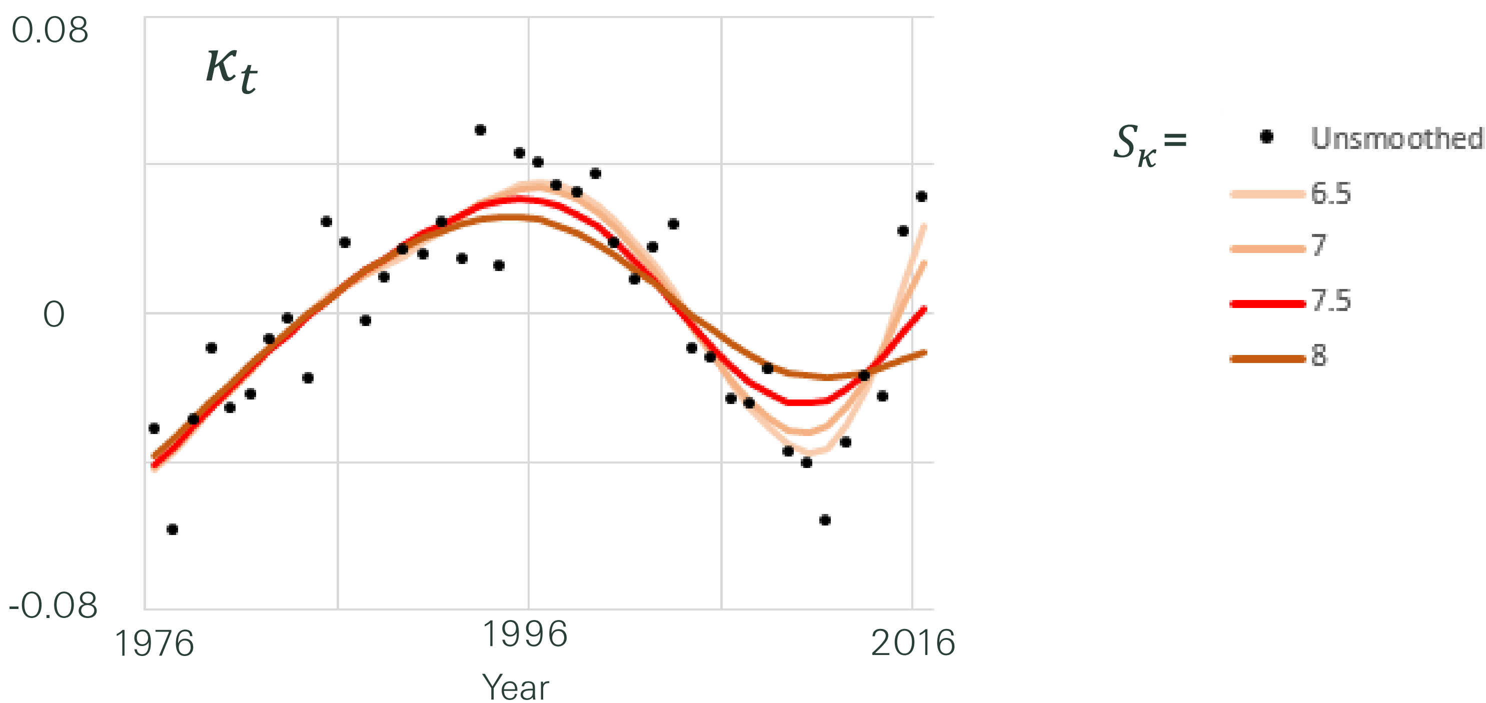
Then find the α_x, β_x, κ_t, and γ_{t-x} which minimise:

$$\text{Objective} = \text{Deviance} + \text{Penalty}(\alpha_x) + \text{Penalty}(\beta_x) + \text{Penalty}(\kappa_t) + \text{Penalty}(\gamma_{t-x})$$

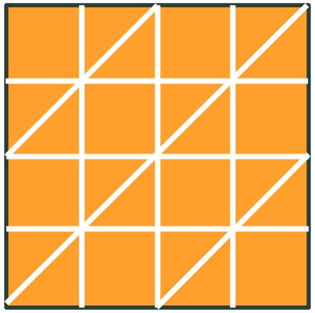
Minimise deviance but penalising solutions which are not ‘smooth’

For example:

$$\text{Penalty}(\kappa_t) = 10^{S_\kappa} \times \sum_t \{(\kappa_t - \kappa_{t-1}) - (\kappa_{t-1} - \kappa_{t-2})\}^2$$

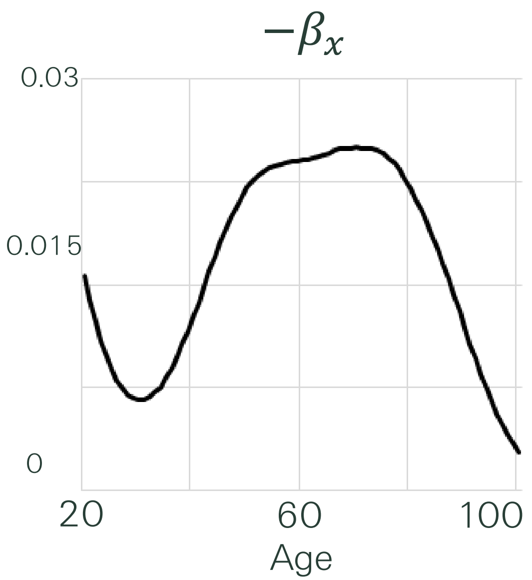


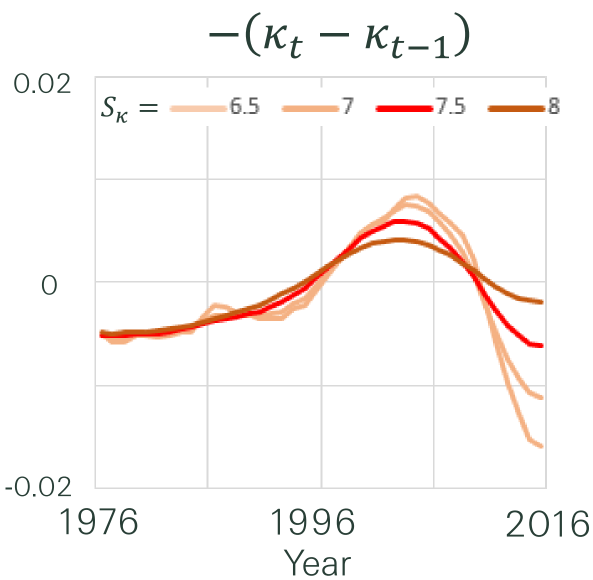
Derived rates of mortality improvement

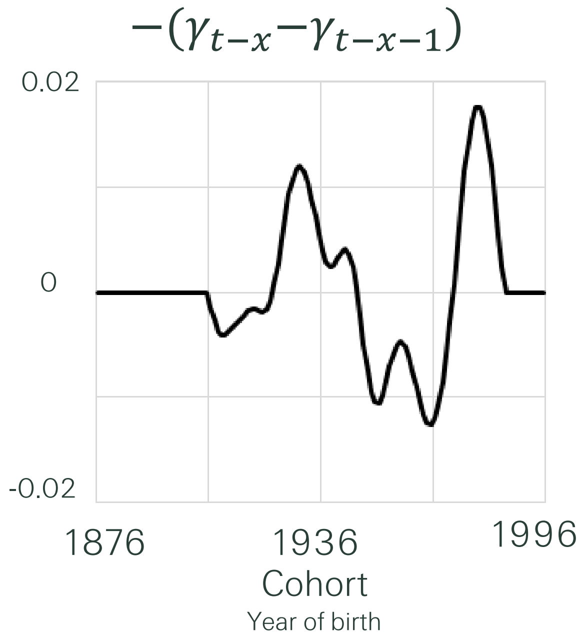


$$\log m_{x,t} = \alpha_x + \beta_x(t-\bar{t}) + \kappa_t + \gamma_{t-x}$$

$$MI^*_{x,t} = \log m_{x,t-1} - \log m_{x,t}$$
$$= -\beta_x - (\kappa_t - \kappa_{t-1}) - (\gamma_{t-x} - \gamma_{t-x-1})$$







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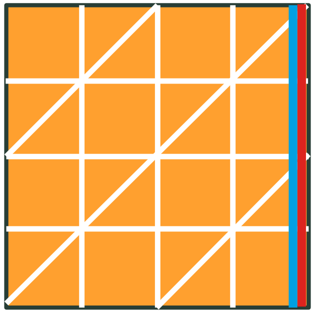
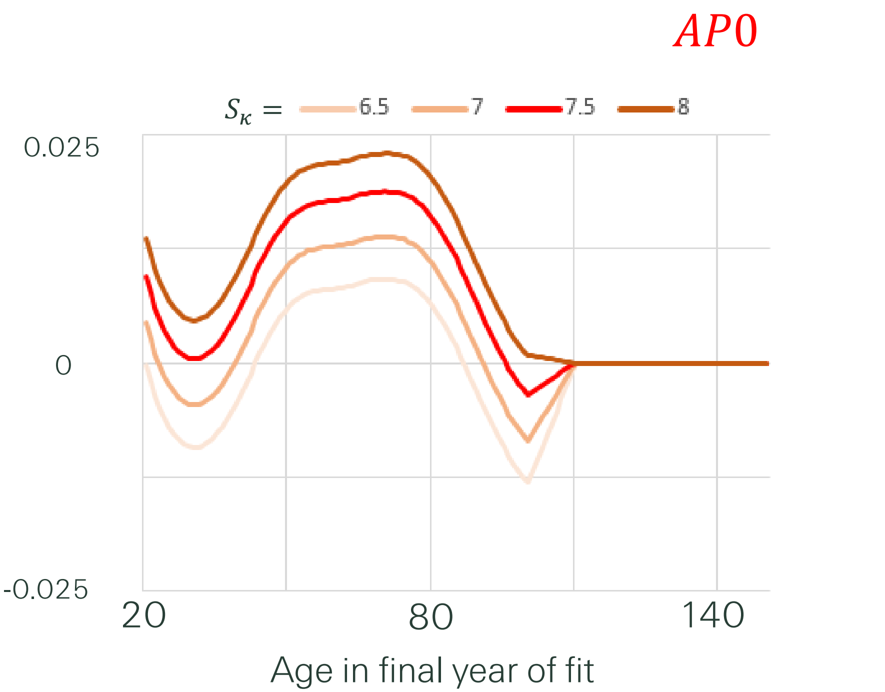
Derived (initial) rates of mortality improvement

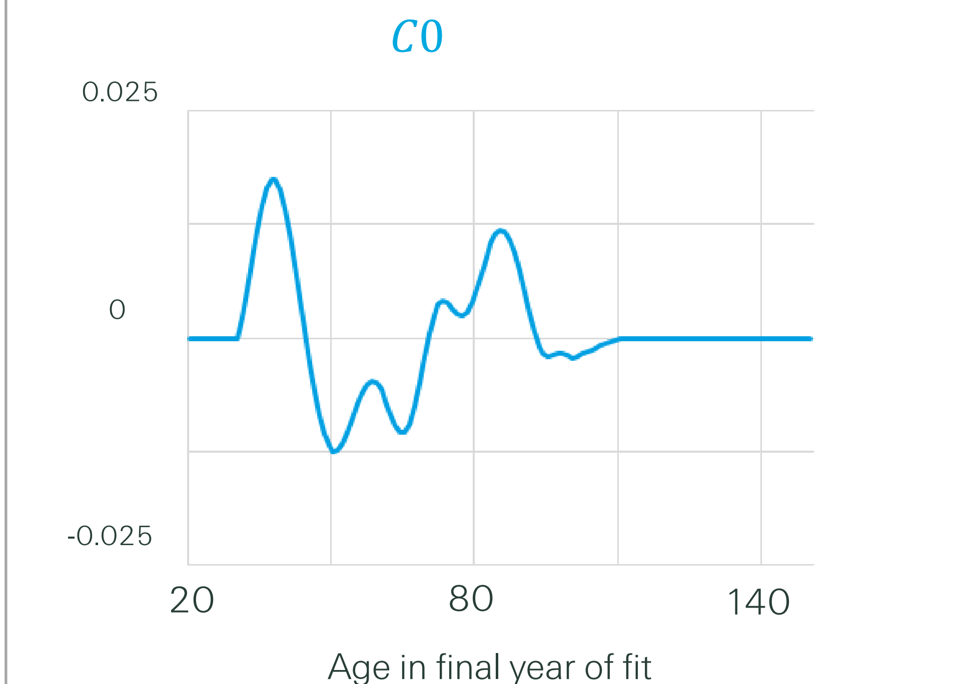


$$\log m_{x,t} = \alpha_x + \beta_x(t - \bar{t}) + \kappa_t + \gamma_{t-x}$$

$$MI^*_{x,t} = \log m_{x,t-1} - \log m_{x,t}$$

$$= -\beta_x - (\kappa_t - \kappa_{t-1}) - (\gamma_{t-x} - \gamma_{t-x-1})$$





Period life expectancy from age 65

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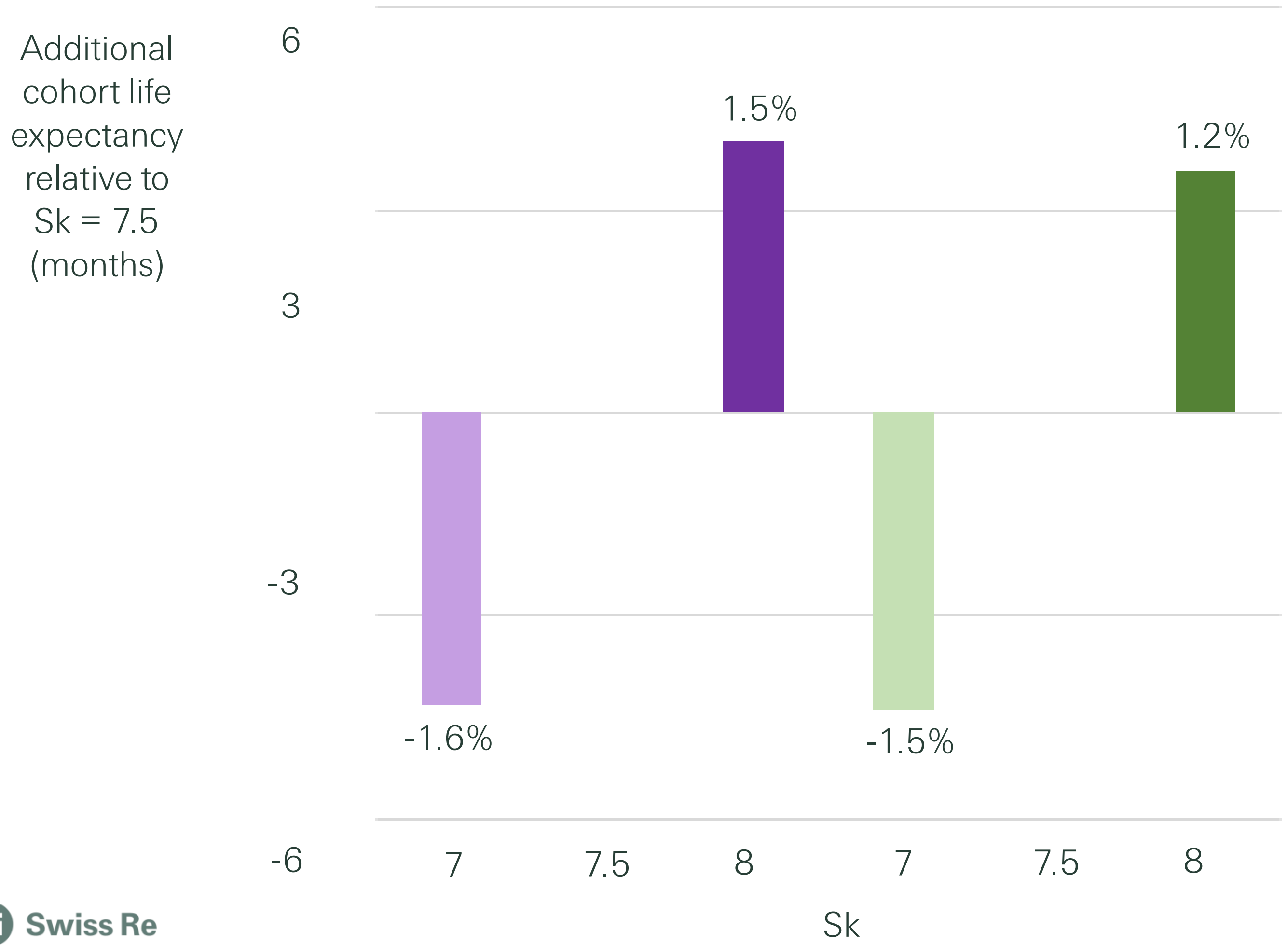
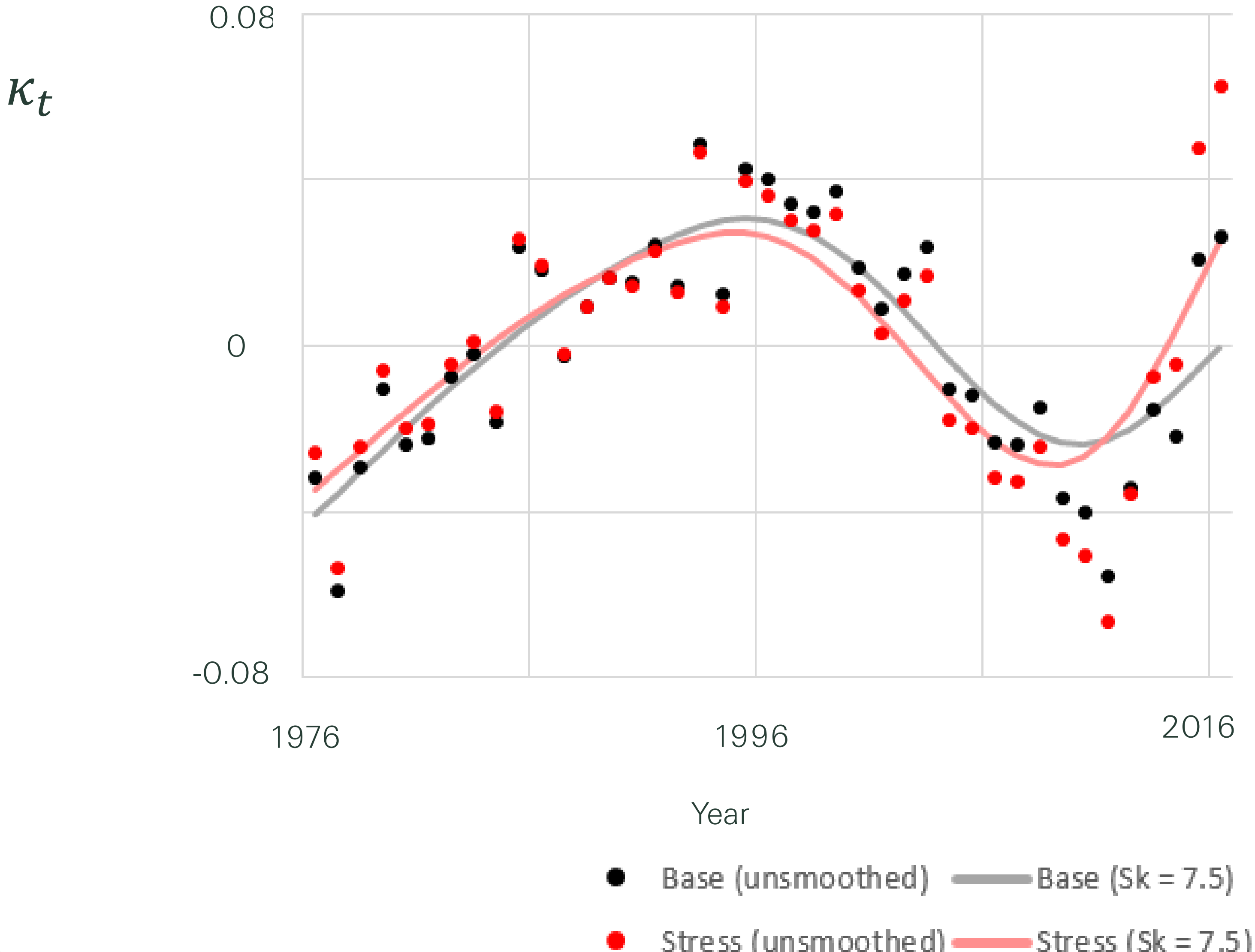
Cohort life expectancy from age 65 relative to Sk = 7.5

	Male			Female		
Sk =	7	7.5	8	7	7.5	8
Cohort life expectancy at 65 =	21.9	22.2	22.6	23.7	24.1	24.4



Cohort life expectancy assuming S2PxA at 1 Jan 2007, long term rate 1.5%

Apply trend stress to final five years



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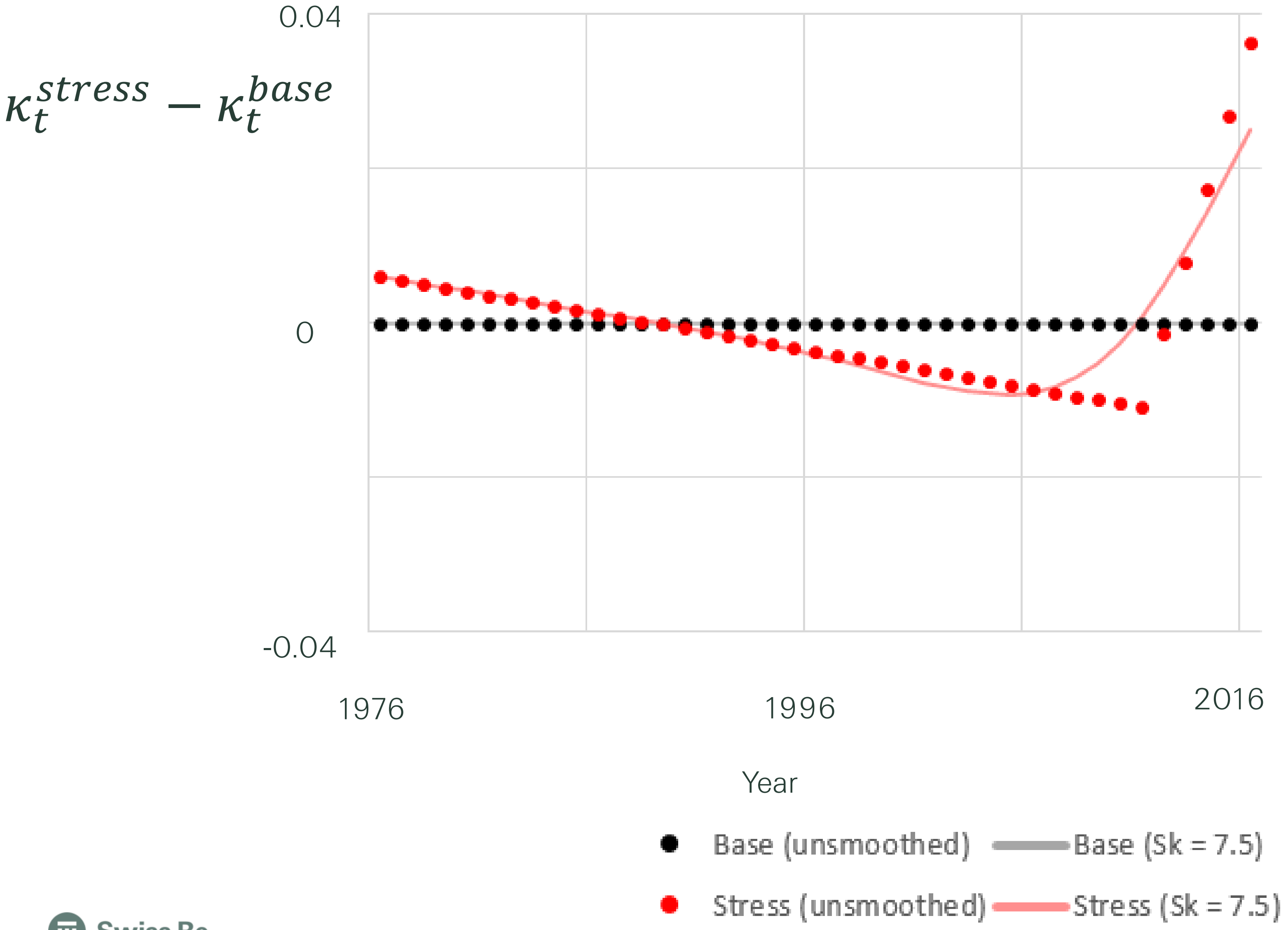
Apply trend stress to final five years



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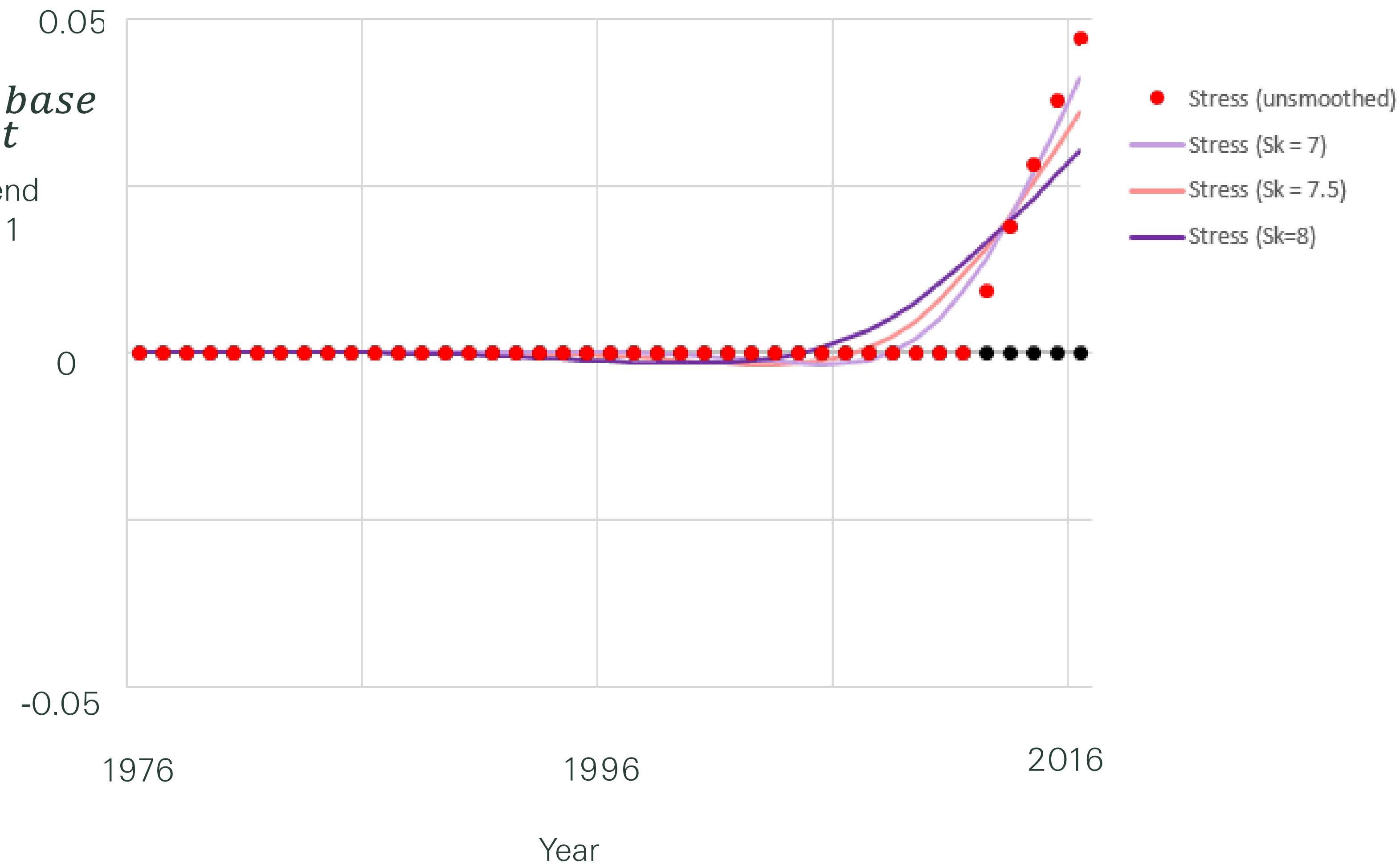
Apply trend stress to final five years

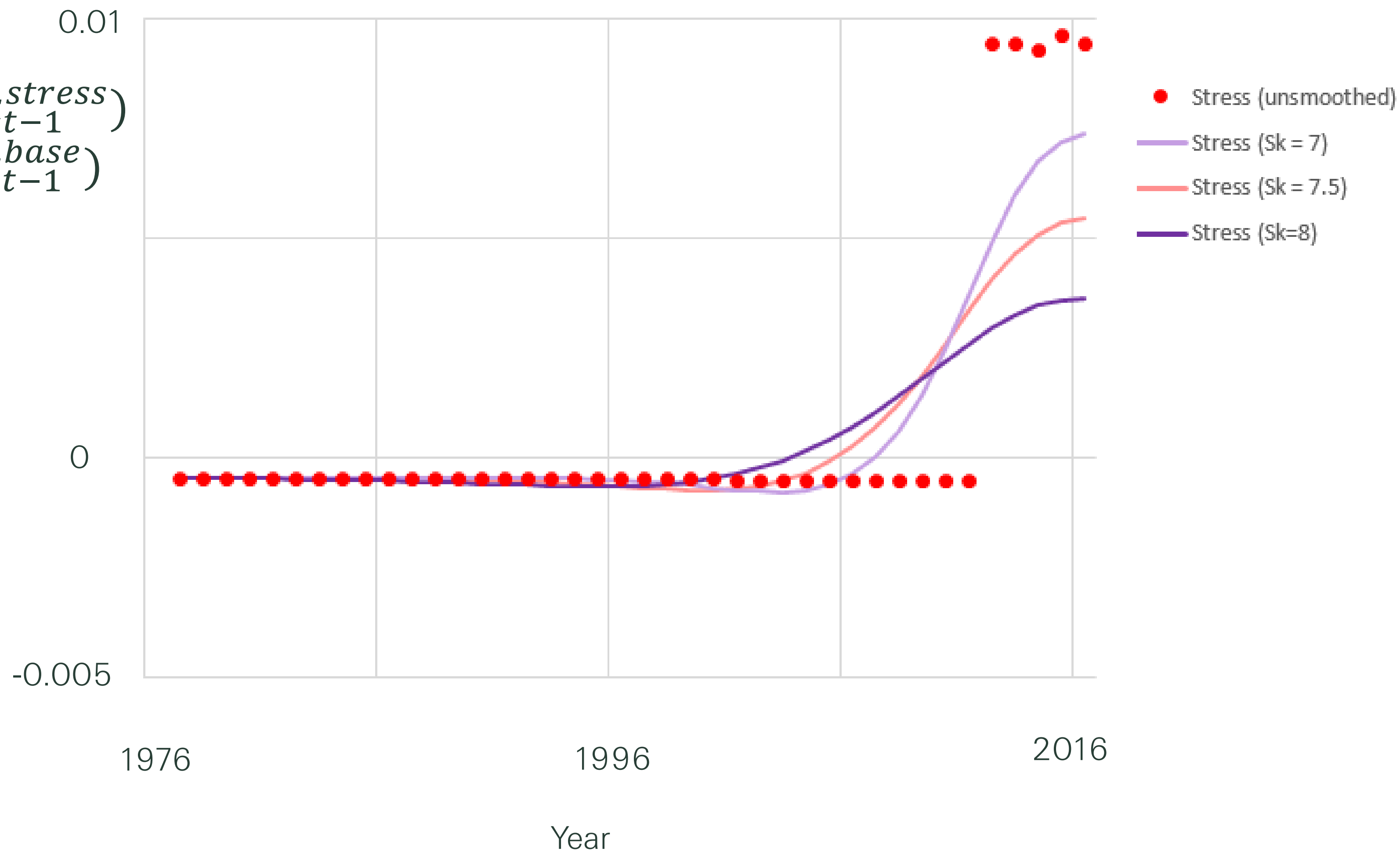
$$\kappa_t^{stress} - \kappa_t^{base}$$

Consistent trend
1976 to 2011

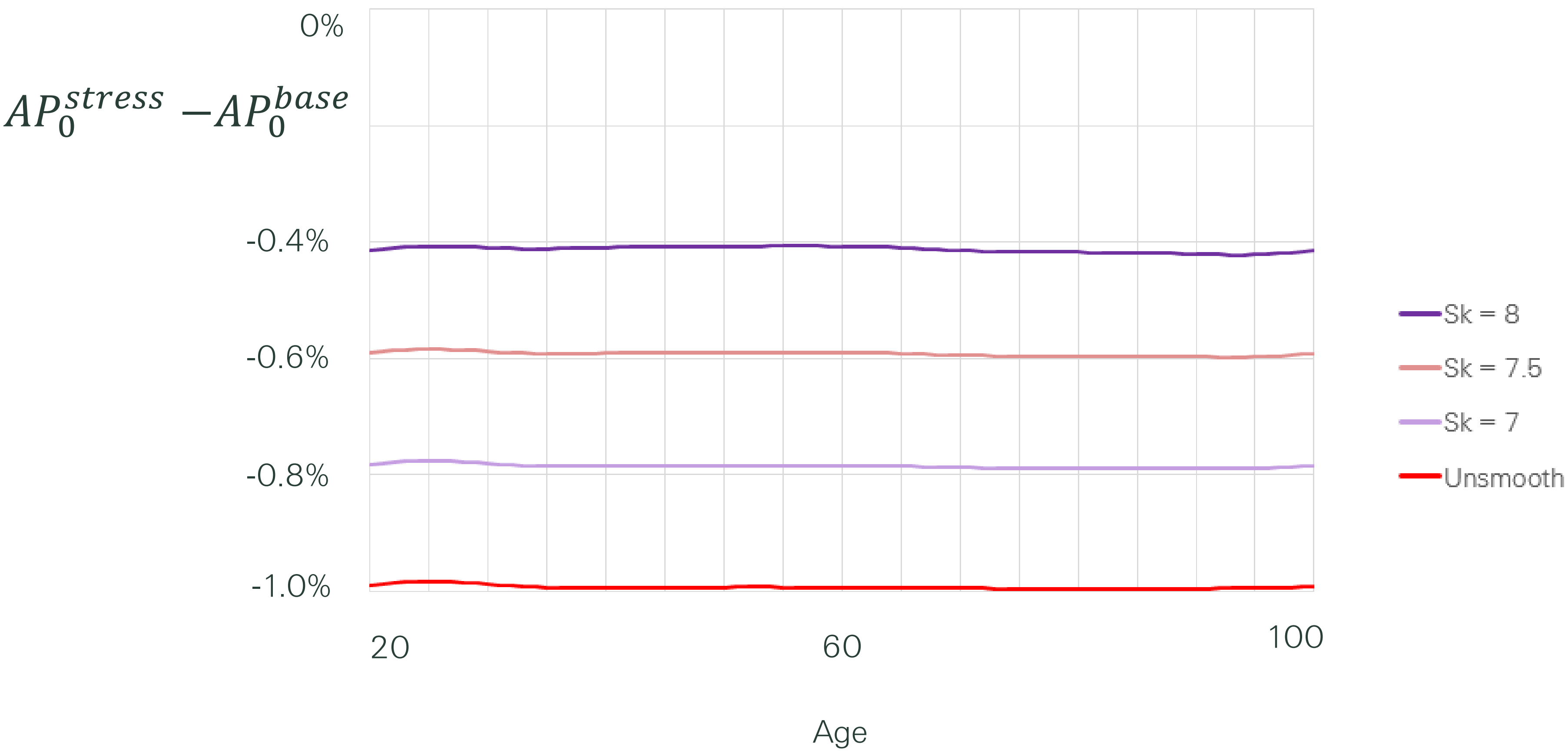


Apply trend stress to final five years

$$-(\kappa_t^{stress} - \kappa_{t-1}^{stress})$$
$$+(\kappa_t^{base} - \kappa_{t-1}^{base})$$

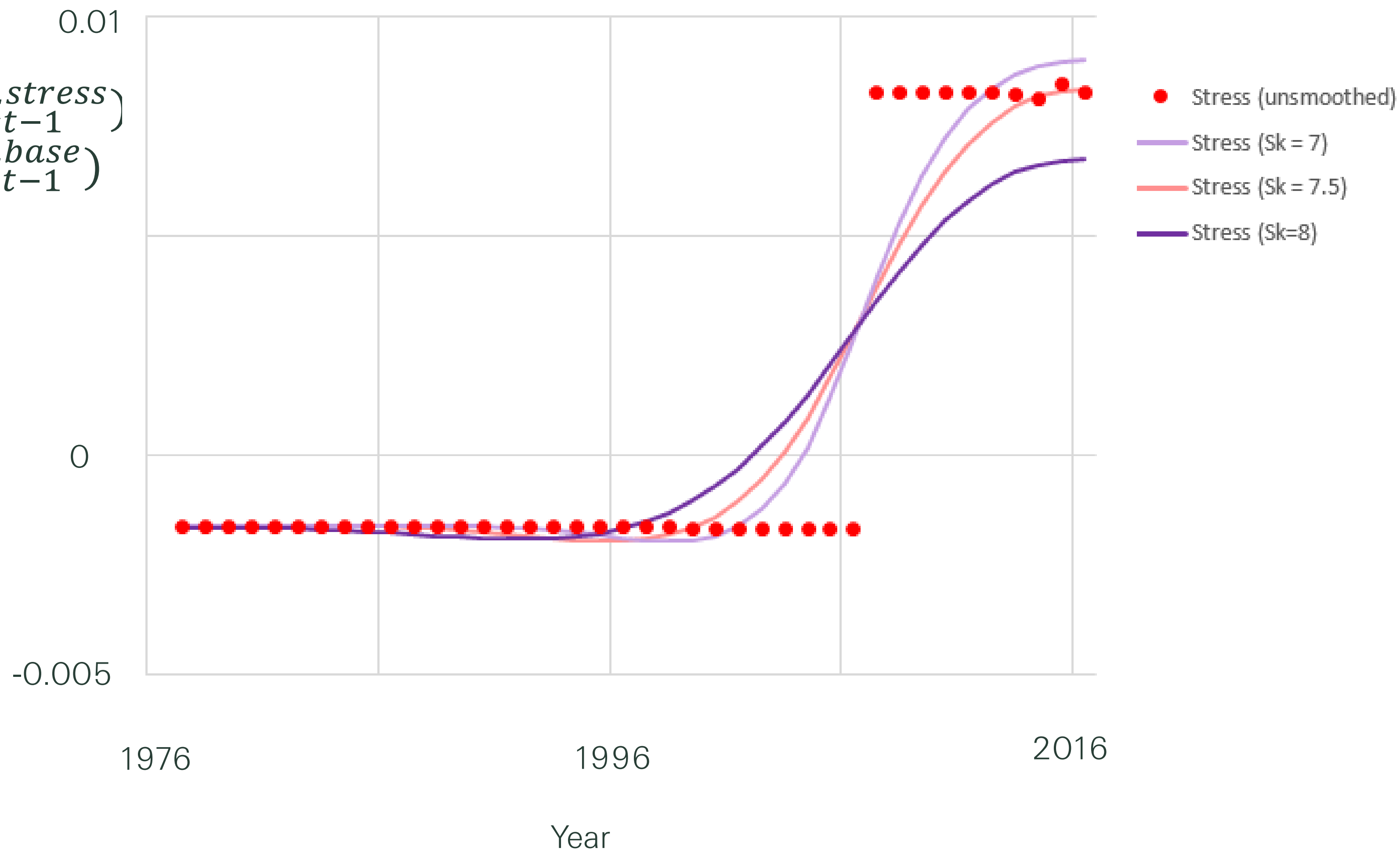


Apply trend stress to final five years

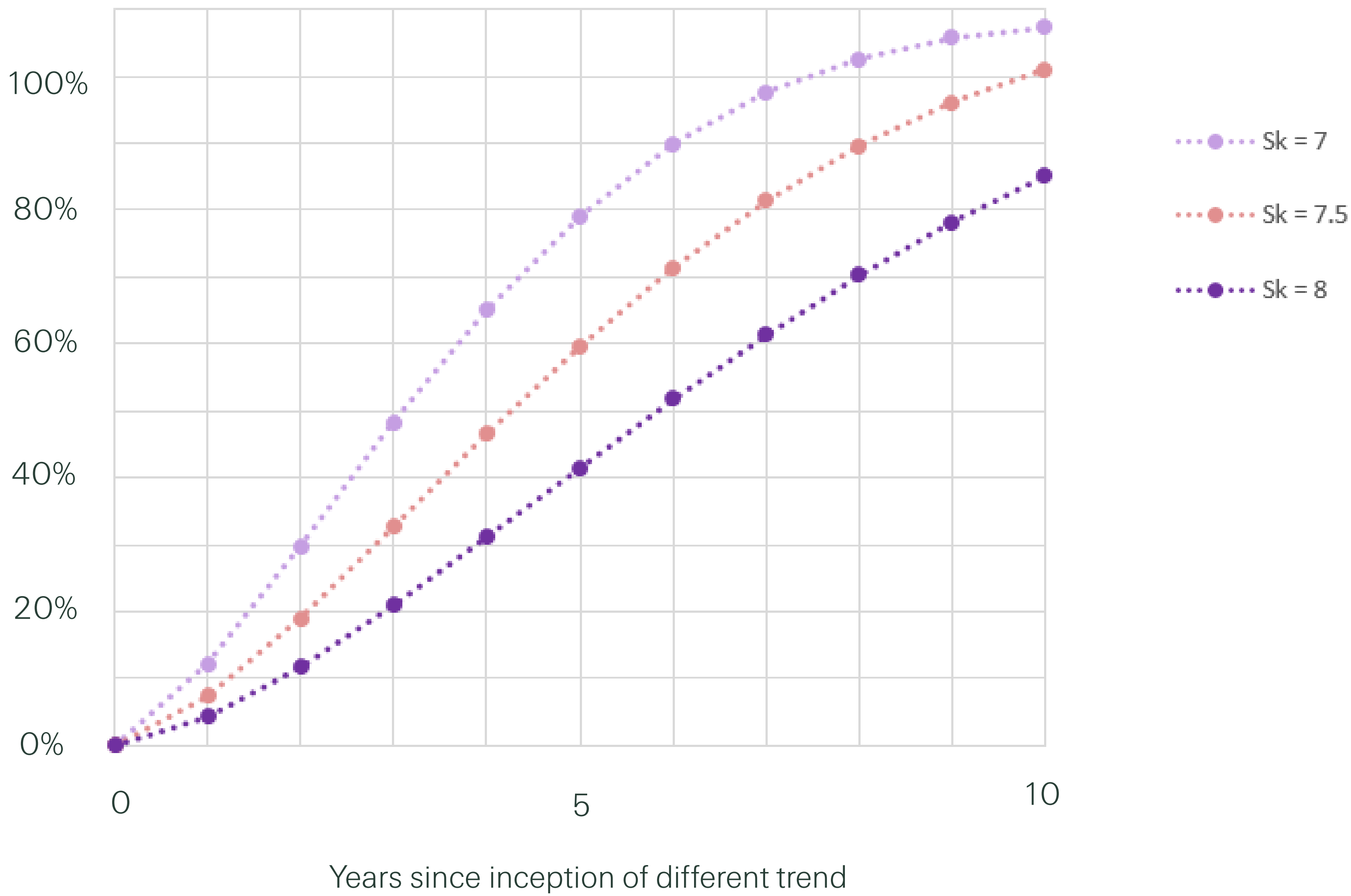


Apply trend stress to final ten years

$$-(\kappa_t^{stress} - \kappa_{t-1}^{stress})$$
$$+(\kappa_t^{base} - \kappa_{t-1}^{base})$$



Amount of new trend which flows into initial improvement by number of years of observation



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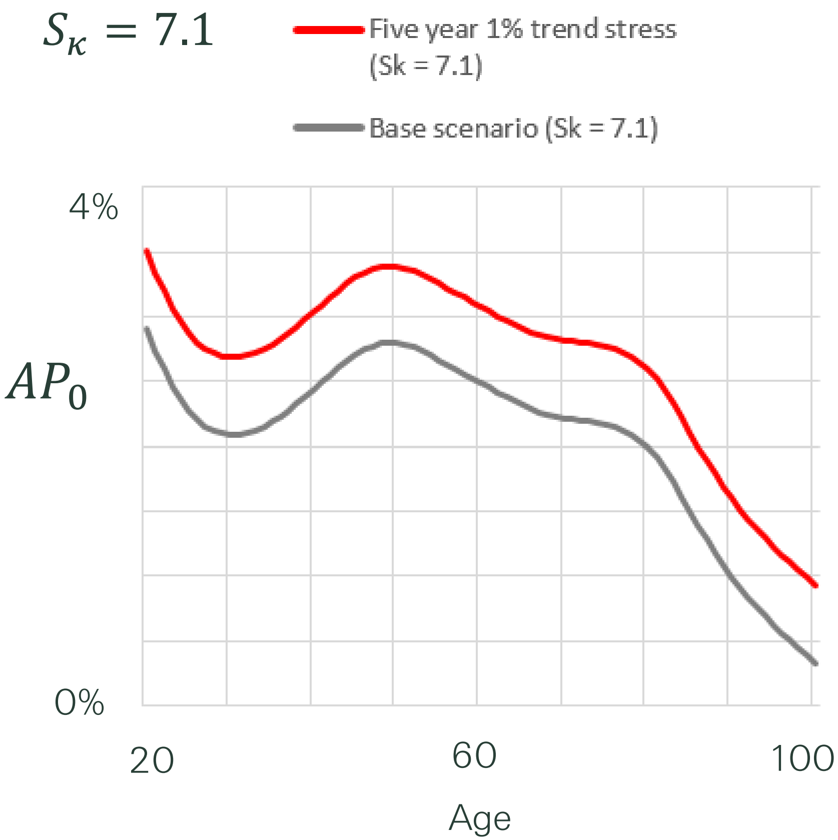
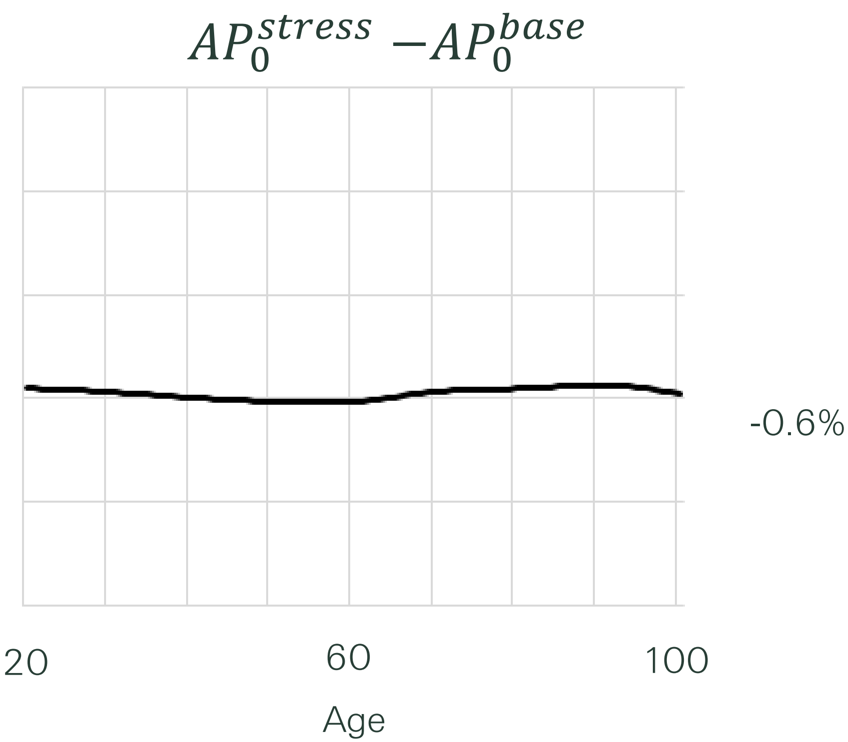
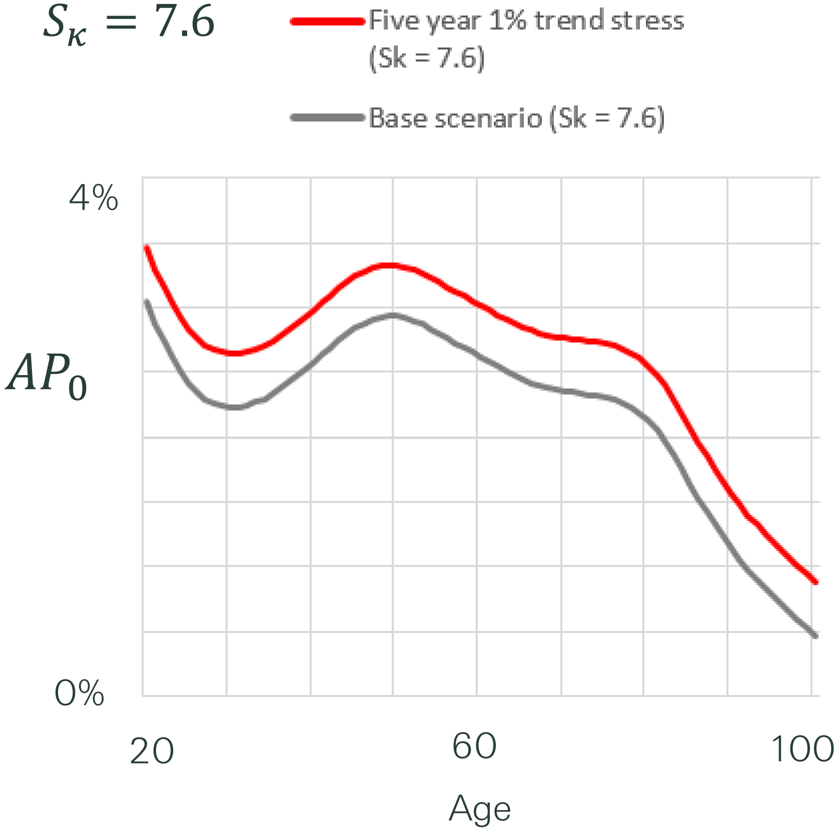
Setting comparable S_κ for a different data set

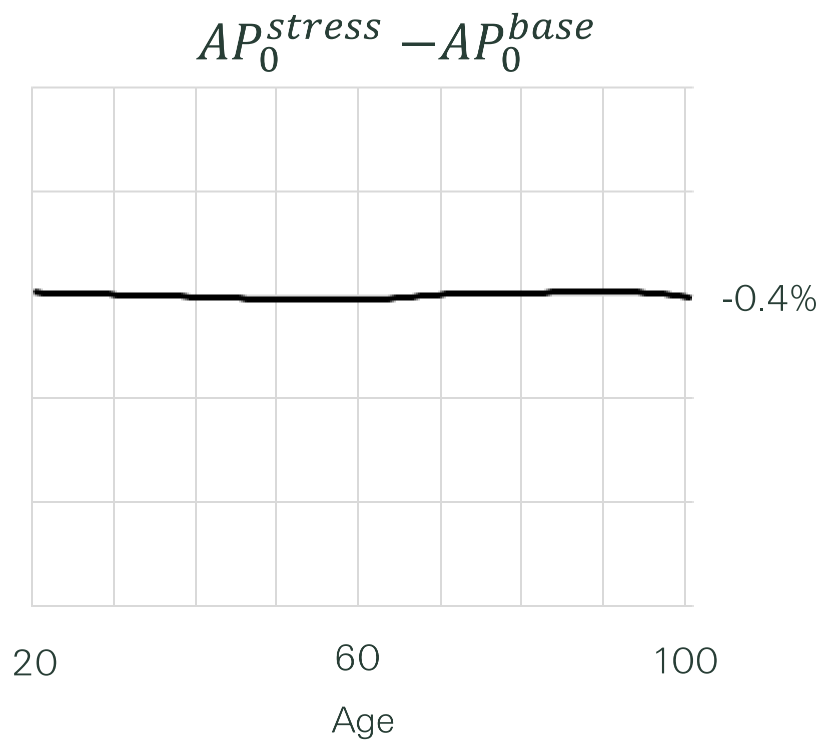




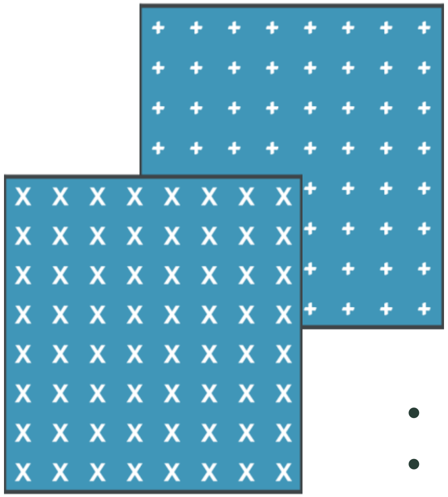
Setting comparable S_κ for a different data set

$S_\kappa = 7.6$





1. Death and exposure data



- Taken from ONS population data for England & Wales
- Counts of deaths and lives (used as a proxy for exposure)
- Crude adjustment applied to reflect known issues in population count data (and using population count data as a proxy for exposure)

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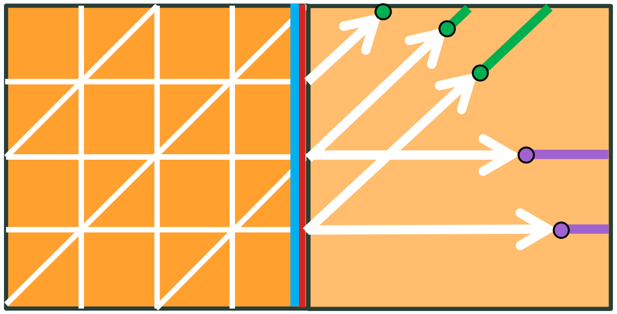
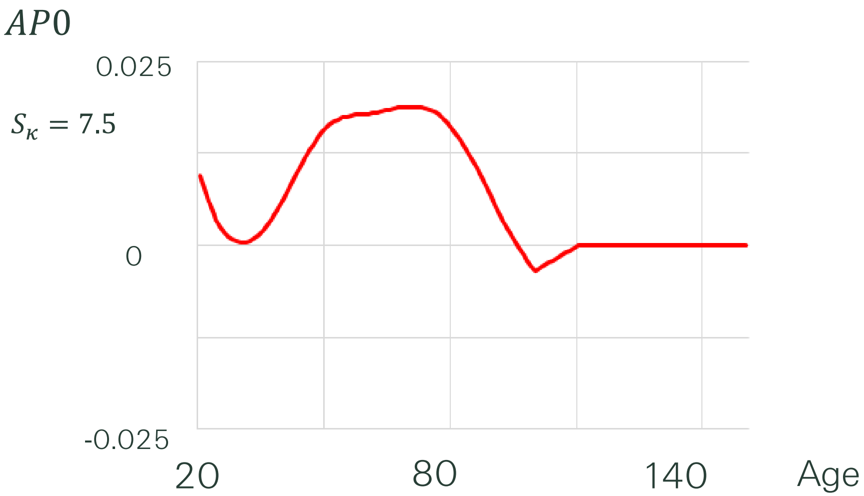
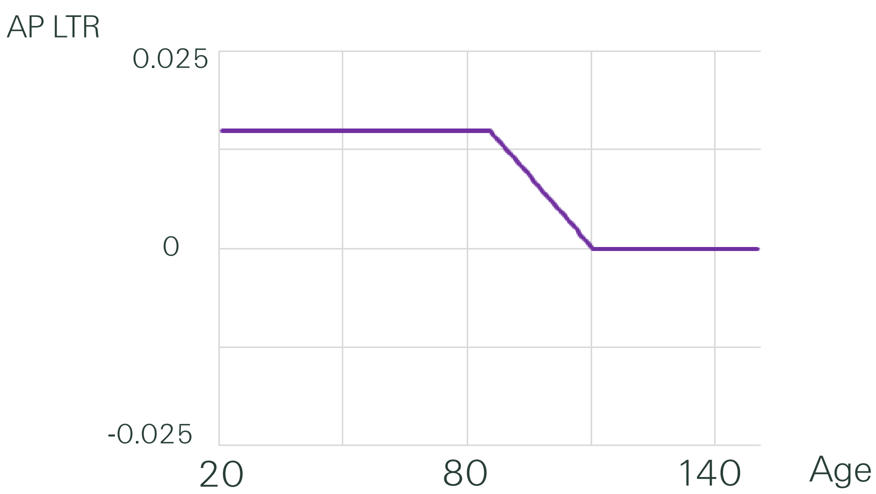
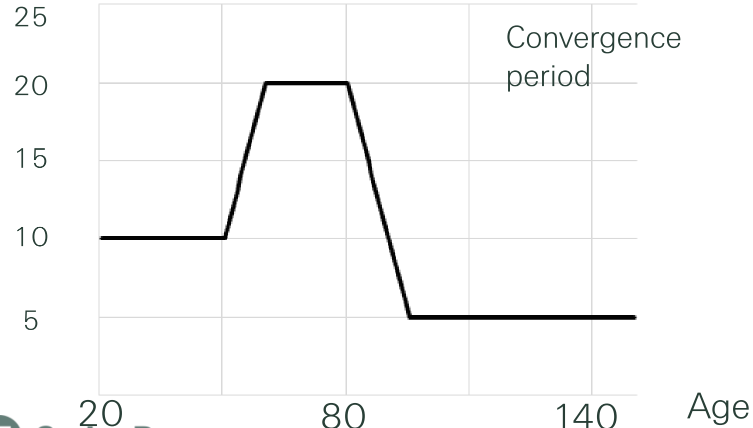
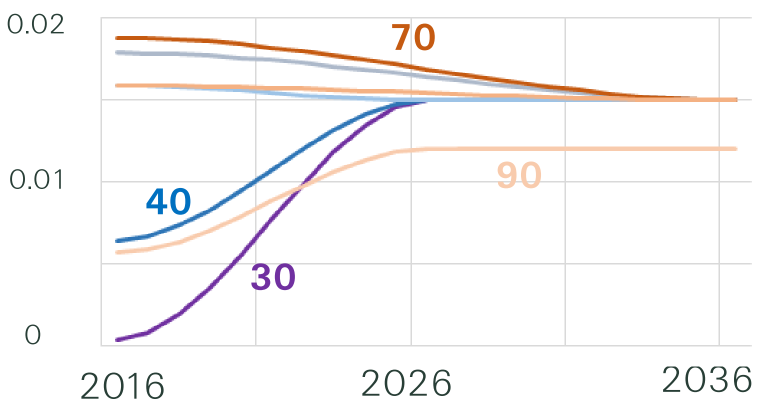
4a. Project from initial rates by age











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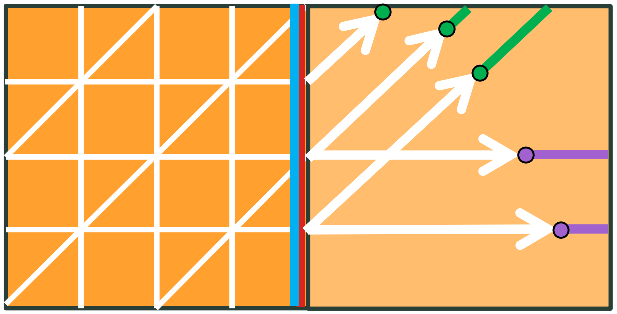
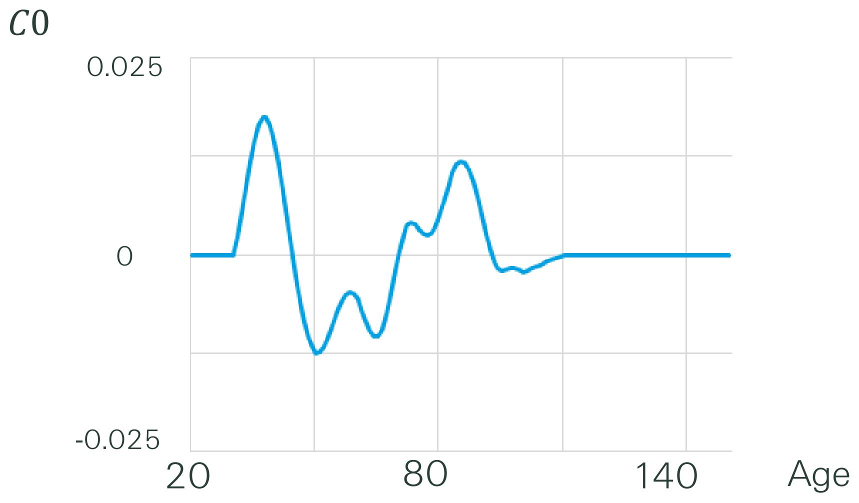
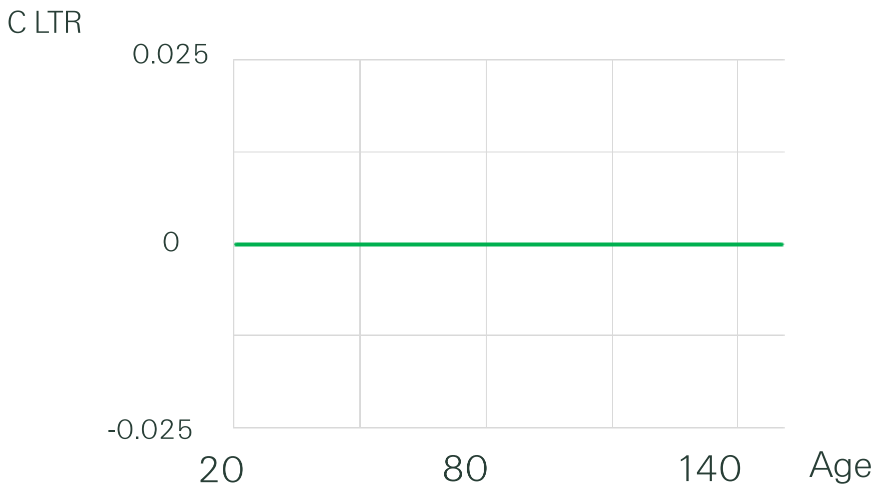
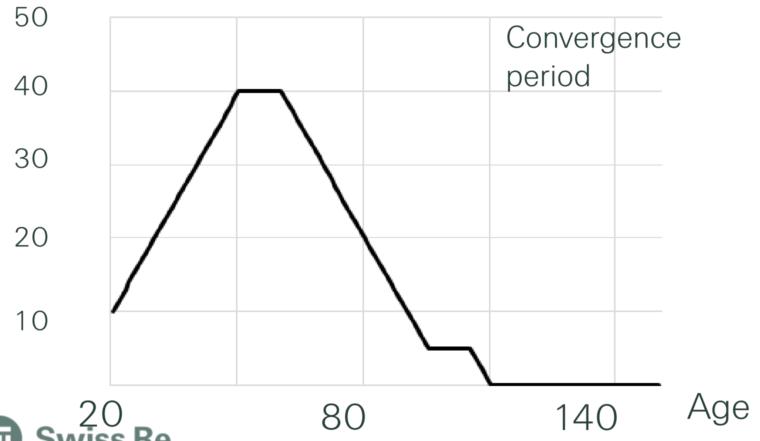
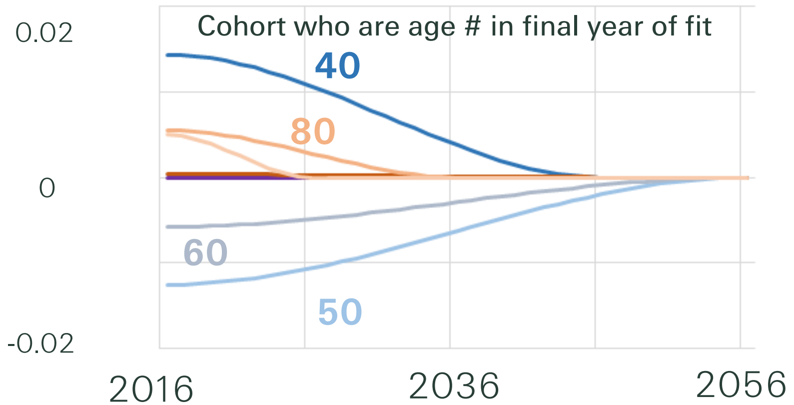
4a. Project from initial rates by cohort











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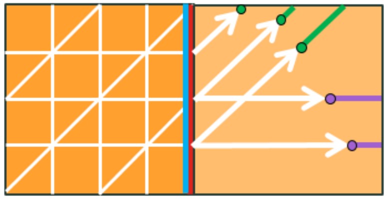
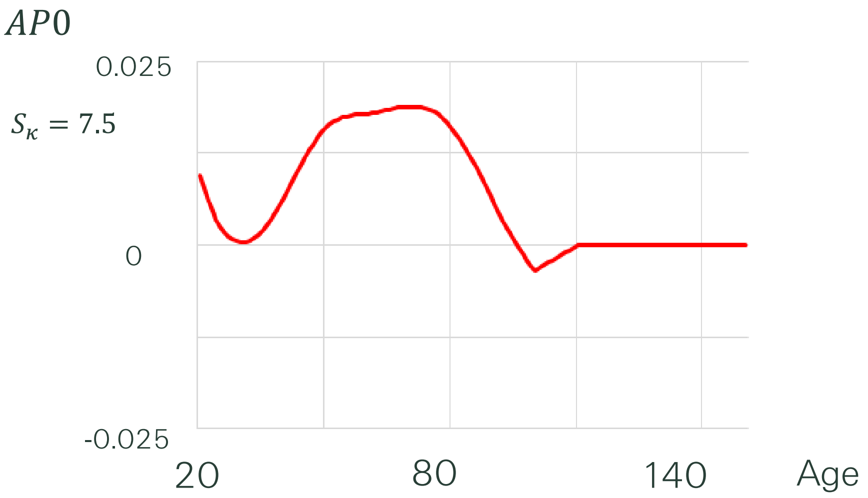
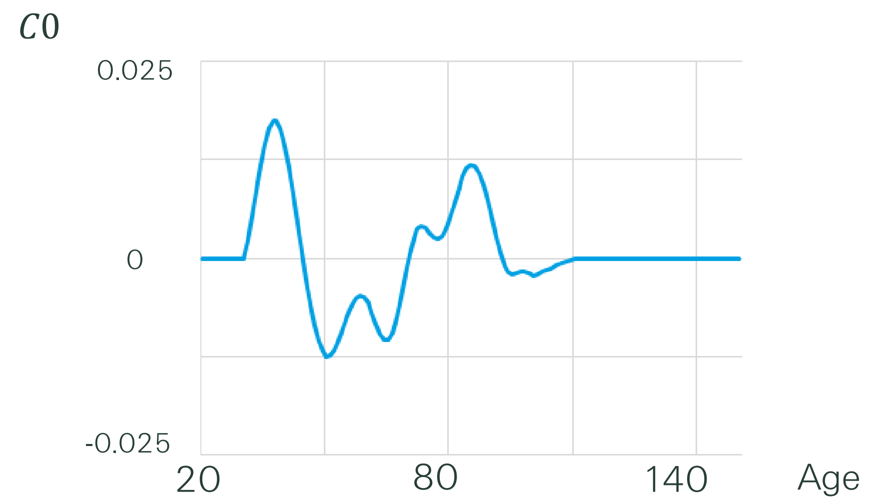
4c. Project from initial rates by cohort and by age

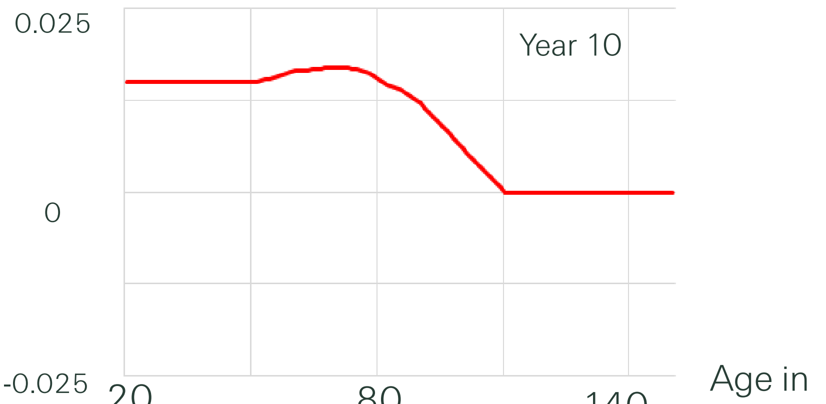


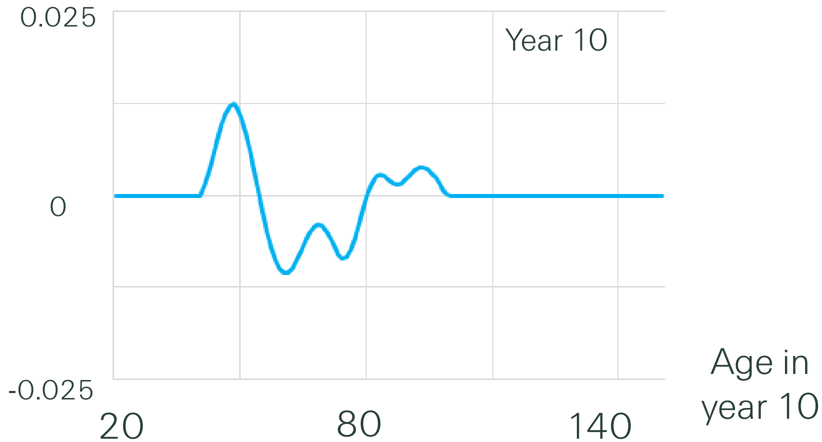
Age-period

Cohort

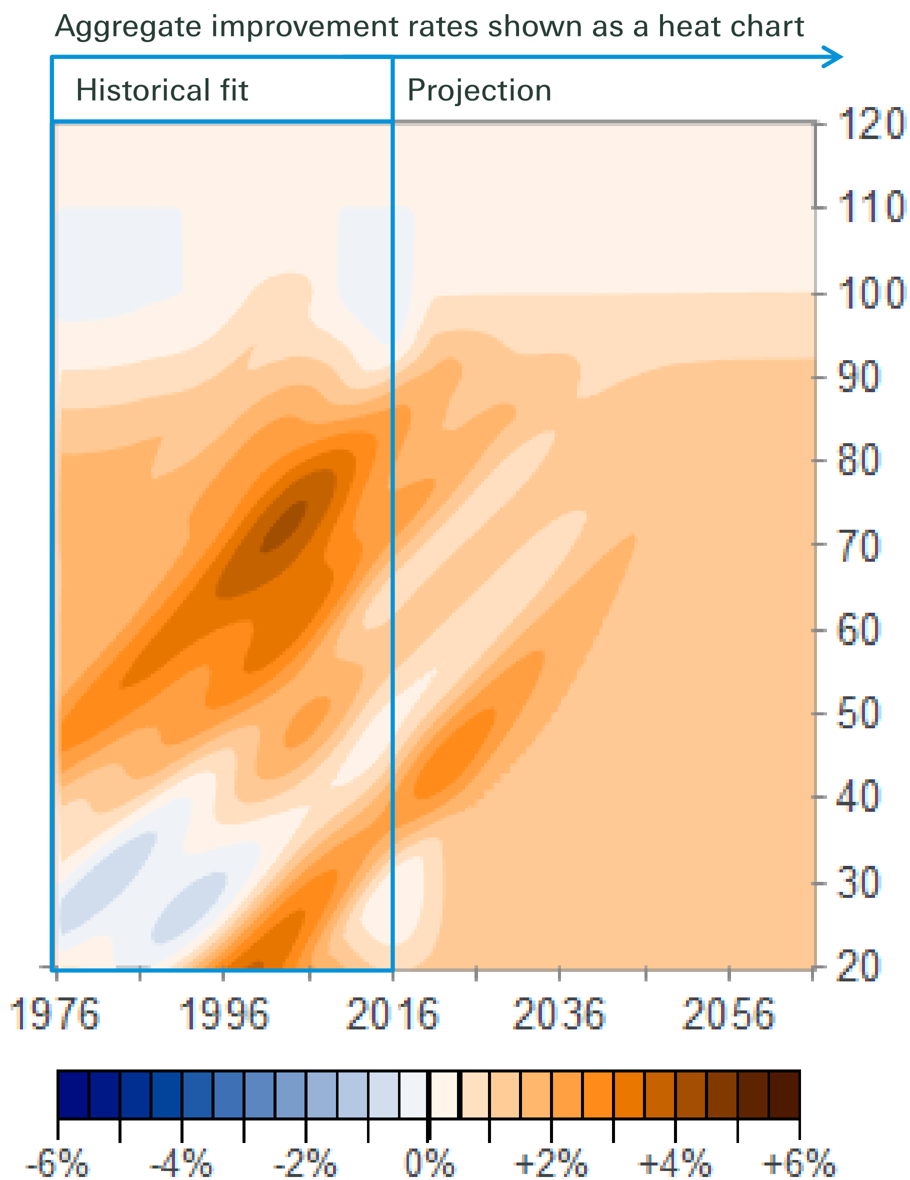




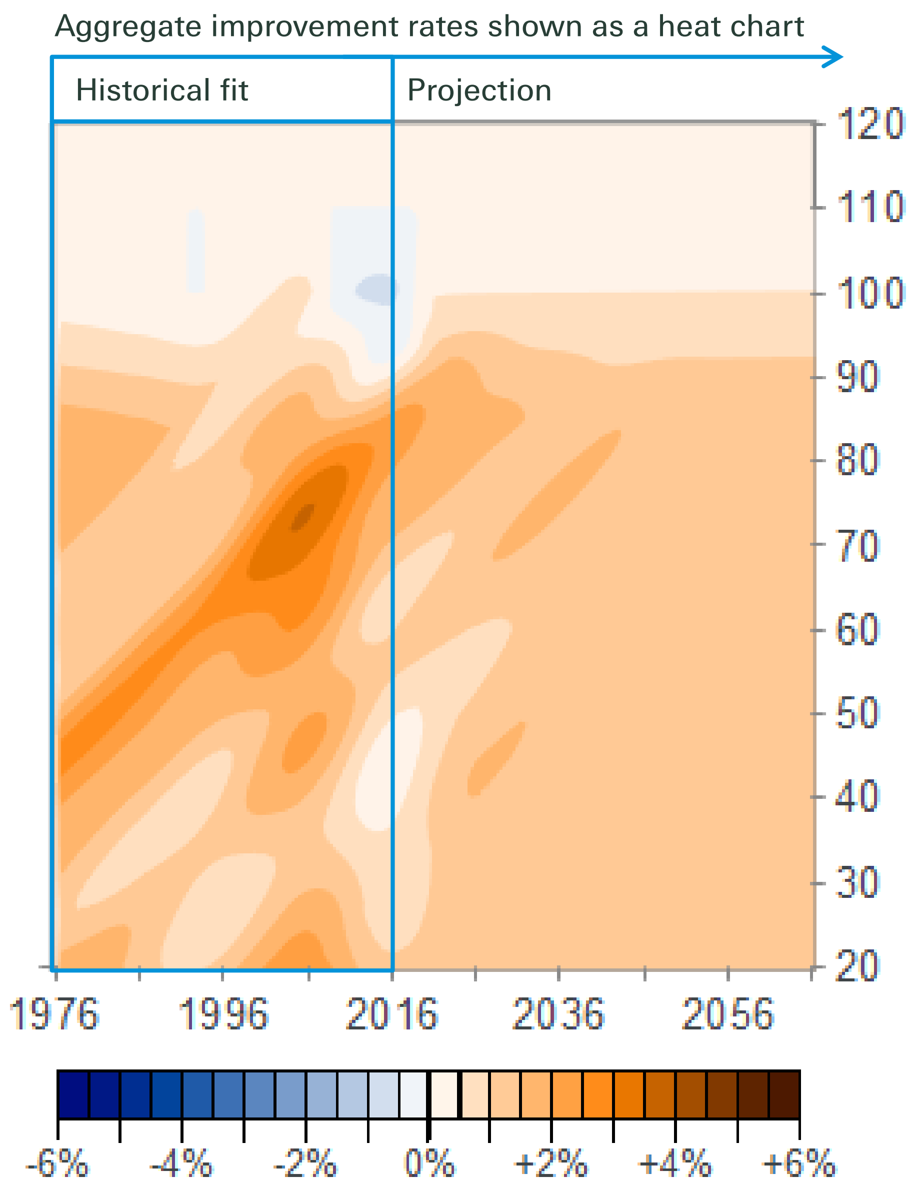




CMI_2016 (1.5%) – heat map males



CMI_2016 (1.5%) – heat map females



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