Risk sharing for public pension schemes

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In context

Ageing population :

- increasing life expectancy
- decreasing birth rate
- *papy boom*

Ratio of pensioners to workers is expected to increase by 46% over the next two decades in Belgium (from 28% in 2017 to 41% in 2037).[1]

Impact of this ratio on the public pension scheme.

[1] The High Council of Finance, 2016

Outline

Pay As You Go pension scheme (PAYG)

Specific pension schemes

Stochastic optimal control

Benefits for specific career paths

Numerical application

PAYG principle

Today workers pay for today pensioners.

PAYG scheme

Incomes from the contributors :

$$A_t \, \pi_t \, \overline{S}_t$$

with

- A : number of contributors
- π : contribution rate
- $\overline{S}$: mean salary

Outcomes for the pensioners :

$$B_t \, \overline{P}_t = B_t \, \delta_t \, \overline{S}_t$$

with

- B : number of pensioners
- δ : global replacement rate
- $\overline{P}$: mean pension

Equilibrium equation

The equilibrium equation of the PAYG scheme is

$$\begin{aligned} \text{Incomes} &= \text{Outcomes} \\ A_t \, \pi_t \, \overline{S}_t &= B_t \, \delta_t \, \overline{S}_t \\ \pi_t &= D_t \, \delta_t \end{aligned}$$

with the dependence ratio

$$D_t = \frac{B_t}{A_t}$$

Equilibrium equation

The equilibrium equation of the PAYG scheme is

$$\begin{aligned} \text{Incomes} &= \text{Outcomes} \\ A_t \, \pi_t \, \overline{S}_t &= B_t \, \delta_t \, \overline{S}_t \\ \pi_t &= D_t \, \delta_t \end{aligned}$$

with the dependence ratio

$$D_t = \frac{B_t}{A_t} \quad \text{risk factor}$$

Automatic balance mechanism

$$\pi_t = D_t \, \delta_t$$

In case of change of the risk factor D_t, how can π_t and δ_t be **automatically adjusted** to maintain the **equilibrium** ...

... while maintaining simultaneously financial sustainability and social adequacy ?

DB and DC schemes

Defined Benefit (DB)	**Defined Contribution (DC)**
$\overline{\delta}$ constant	$\overline{\pi}$ constant
$\pi_t = D_t \, \overline{\delta}$	$\delta_t = \dfrac{\overline{\pi}}{D_t}$
Demographic risk borne by the **contributors**	Demographic risk borne by the **pensioners**

An intermediate scheme : the Musgrave rule

Replacement rate **net of contribution** M constant

$$M = \frac{\overline{P}_t}{S_t\,(1-\pi_t)} = \frac{\delta_t}{1-\pi_t}$$

$$\begin{cases} \delta_t = \dfrac{M}{1 + M\,D_t} \\ \pi_t = \dfrac{M\,D_t}{1 + M\,D_t} \end{cases}$$

Risk shared by the contributors and the pensioners

Optimal criteria

Optimal risk sharing providing joint **stability** of π_t and δ_t around fixed targets $\overline{\pi}$ and $\overline{\delta}$.

Optimization and its loss function

$$\min_{\delta_s,\pi_s} \mathbb{E}\left[\int_0^T \left((1-\rho_s)\left(\frac{\delta_s}{\overline{\delta}}-1\right)^2+\rho_s\left(\frac{\pi_s}{\overline{\pi}}-1\right)^2\right)ds)\right]^2$$

with fix targets $\overline{\delta}$, $\overline{\pi}$ and a given weight process $\rho_s \in [0,1]$

The dependence ratio process follows a geometric Brownian motion

$$\frac{dD_t}{D_t} = \mu\,dt + \sigma\,dW_t$$

where W_t is a standard Brownian motion.

[2] A. Cairns, 2000

Stochastic optimal control

We use the PAYG equilibrium equation

$$\pi_t = D_t \, \delta_t \, .$$

- The loss function is

$$(1-\rho_t)\left(\frac{\delta_t}{\overline{\delta}} - 1\right)^2 + \rho_t \left(\frac{D_t \, \delta_t}{\overline{\pi}} - 1\right)^2 \, .$$

- The state variable is

$$D \, .$$

- The control variable is

$$\delta \, .$$

Optimal solutions

By applying the stochastic optimal control theory (HJB equation), we obtain

$$\delta_t^\star = \frac{(1-\rho_t)\,\frac{1}{\overline{\delta}} + \rho_t\,\frac{D_t}{\overline{\pi}}}{(1-\rho_t)\,\frac{1}{\overline{\delta}^2} + \rho_t\,\frac{D_t^2}{\overline{\pi}^2}}$$

$$\pi_t^\star = D_t\,\delta^\star(t)$$

The obtained result does not depend on the type of the process D_t.

This result can be directly obtained by optimizing the loss function.

Optimal solutions

Extreme DB and DC schemes

$$\delta_t^\star = \frac{(1-\rho_t)\,\frac{1}{\overline{\delta}} + \rho_t\,\frac{D_t}{\overline{\pi}}}{(1-\rho_t)\,\frac{1}{\overline{\delta}^2} + \rho_t\,\frac{D_t^2}{\overline{\pi}^2}}$$

$$\pi_t^\star = D_t\,\delta^\star(t)$$

DB : $\rho_t = 0$ $\begin{cases} \delta_t^\star = \overline{\delta} \\ \pi_t^\star = D_t\,\overline{\delta} \end{cases}$ DC : $\rho_t = 1$ $\begin{cases} \delta_t^\star = \dfrac{\overline{\pi}}{D_t} \\ \pi_t^\star = \overline{\pi} \end{cases}$

Calibration of the targets

The targets $\overline{\pi}$ and $\overline{\delta}$ are determined according following constraints

$$\begin{aligned} \delta^\star_{t_0} &= \delta_0 && \text{(initialization)} \\ \overline{\pi} &= \overline{\delta}\, D_\infty && \text{(PAYGO equation).} \end{aligned}$$

For a constant ρ, we obtain

$$\begin{aligned} \overline{\pi} &= \delta_0 \frac{(1-\rho)\, D_\infty^2 + \rho\, D_0^2}{(1-\rho)\, D_\infty + \rho\, D_0} \\ \overline{\delta} &= \delta_0 \frac{(1-\rho) + \rho\, \dfrac{D_0^2}{D_\infty^2}}{(1-\rho) + \rho\, \dfrac{D_0}{D_\infty}}\,. \end{aligned}$$

Benefits for specific career paths

Individual δ^i depends on the specific career profile $\{S_x^i\}$.

The **points system** is used to determine the pension P^i according to the career of each affiliate.

Points system

Benefits :

$$P_t^i = \Pi^i \, v_t$$

Number of points :

$$\Pi^i = \sum_{x=x_0}^{x_r-1} \frac{S_x^i}{\overline{S}}$$

constant mean salary over time $\overline{S}$

Value of the point :

$$\begin{aligned} v_t &= \delta_t \, \overline{S} \, \frac{1}{\Pi_{ref}} \\ &= \delta_t \, \overline{S} \, \frac{1}{x_r - x_0} \end{aligned}$$

Points system

Benefits :

$$\begin{aligned} P_t^i &= \Pi^i \, v_t \\ &= \frac{1}{x_r - x_0} \, \delta_t \sum S_x^i \end{aligned}$$

Individual replacement rate :

$$\begin{aligned} \delta_t^i &= \frac{P_t^i}{S_{x_{r-1}}^i} \\ &= \frac{1}{x_r - x_0} \, \frac{1}{S_{x_{r-1}}^i} \, \delta_t \sum S_x^i \end{aligned}$$

Another risk sharing

With the proposed risk sharing and points system, we can not define a **DB system on final salary** with the same replacement rate for everyone independently of the career profile. In order to obtain a DB system on final salary, we propose a new risk sharing :

$$\gamma\, DB + (1-\gamma)\, DC \qquad \text{with}\, \gamma \in [0,1]\ .$$

Another risk sharing

Extreme DB and DC schemes

$$\gamma\, DB + (1-\gamma)\, DC \qquad \text{with}\, \gamma \in [0,1]$$

DB ($\gamma = 1$)

$$\begin{cases} \pi_t = D_t\,\overline{\delta} \\ \delta_t^i = \delta_t = \overline{\delta} \end{cases}$$

DC ($\gamma = 0$)

$$\begin{cases} \pi_t = \overline{\pi} \\ \delta_t = \dfrac{\overline{\pi}}{D_t} \\ \delta_t^i = \frac{1}{x_r - x_0}\,\frac{1}{S_{x_r}^i}\,\frac{\overline{\pi}}{D_t}\,\sum S_x^i \end{cases}$$

Another risk sharing

$$\gamma\, DB + (1-\gamma)\, DC \qquad \text{with}\, \gamma \in [0,1]$$

Risk sharing

$$\begin{cases} \pi_t = \gamma\, D_t\, \overline{\delta} + (1-\gamma)\, \overline{\pi} \\ \delta_t = \gamma\, \overline{\delta} + (1-\gamma)\, \frac{\overline{\pi}}{D_t} \\ \delta_t^i = \gamma\, \overline{\delta} + (1-\gamma) \frac{1}{x_r - x_0}\, \frac{1}{S_{x_r}^i}\, \frac{\overline{\pi}}{D_t}\, \sum S_x^i \end{cases}$$

Pay As You Go pension scheme (PAYG)

Specific pension schemes

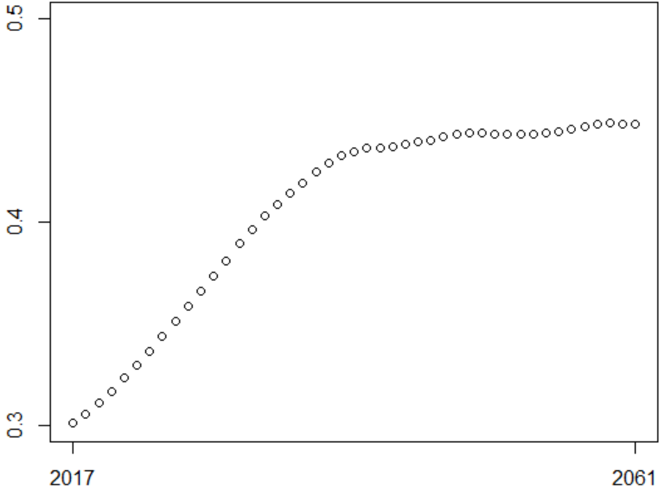
Stochastic optimal control

Benefits for specific career paths

Numerical application

Dependence ratio[3]

Complete career from $x_0 = 20$ years to $x_r = 65$ years



[3]Federal Planning Bureau, 2014

Dependence ratio process

The dependence ratio is a mean reverting process and follows a lognormal distribution : the **Black-Karasinski model**

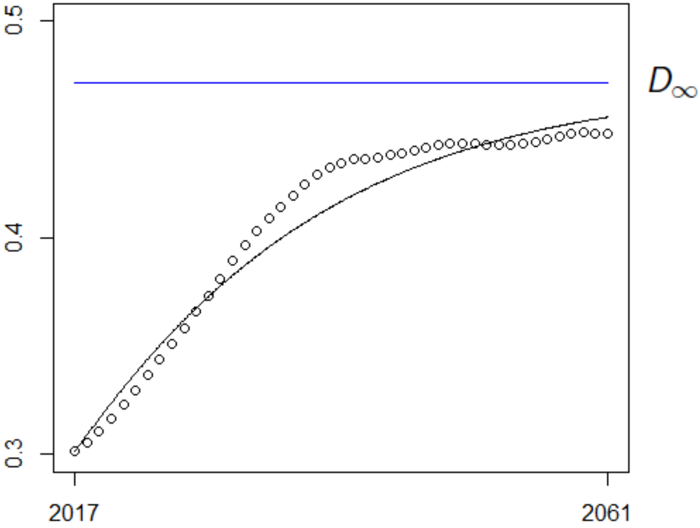
$$d \ln D(t) = \alpha \left(\ln D_\infty - \ln D(t) \right) \, dt + \sigma \, dW(t)$$

where $\alpha > 0$, $\sigma > 0$ and W_t is a standard Brownian motion.

Calibration using least square regression provides the parameters

$$\alpha = 0.059 \, , \quad D_\infty = 0.47 \quad \text{and} \quad \sigma = 0.0046 \; .$$

Dependence ratio process



Initial conditions

The initialisation of our model in $t_0 = 2017$:

- Dependence ratio : $D_0 = 30\%$
- Contribution rate : $\pi_0 = 15\%$
- Replacement rate : $\delta_0 = 50\%$
- Net replacement rate : $M = 59\%$

Risk sharing under stochastic optimal control

Remember, the loss function to minimize is

$$(1-\rho_t)\left(\frac{\delta_t}{\overline{\delta}}-1\right)^2+\rho_t\left(\frac{D_t\,\delta_t}{\overline{\pi}}-1\right)^2 \ .$$

We simulate scenarios with

$\rho = \{0, 0.25, 0.5, 0.75, 1\}$ and the Musgrave rule.

Risk sharing under stochastic optimal control

Six scenarios

$$(1-\rho_t)\left(\frac{\delta_t}{\overline{\delta}}-1\right)^2+\rho_t\left(\frac{D_t\,\delta_t}{\overline{\pi}}-1\right)^2$$

Simulated scenarios

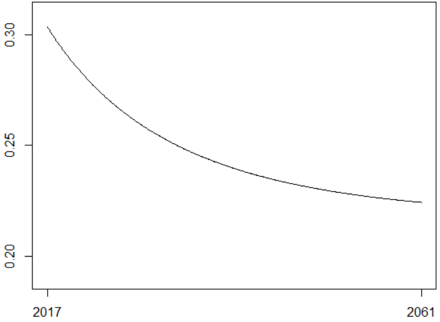
scheme	ρ_t	$\overline{\pi}$	$\overline{\delta}$
DB	0	24%	50%
risk sharing	0.25	22%	47%
Musgrave	$\tilde{\rho}_t$	22%	46%
risk sharing	0.5	20%	43%
risk sharing	0.75	18%	38%
DC	1	15%	32%

Risk sharing under stochastic optimal control

Musgrave rule

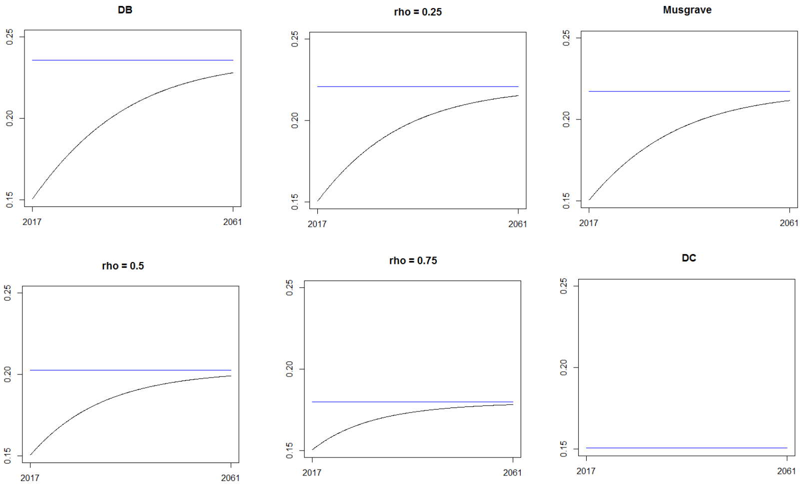
The weight process is

$$\tilde{\rho}_t = \frac{\frac{1}{\overline{\overline{\delta}}}\left(1 - \frac{M}{\overline{\overline{\delta}}} + M\ D_t\right)}{\frac{1}{\overline{\overline{\delta}}}\left(1 - \frac{M}{\overline{\overline{\delta}}} + M\,D_t\right) - \frac{D_t}{\overline{\overline{\pi}}}\left(1 - \frac{M\,D_t}{\overline{\overline{\pi}}} + M\,D_t\right)} .$$



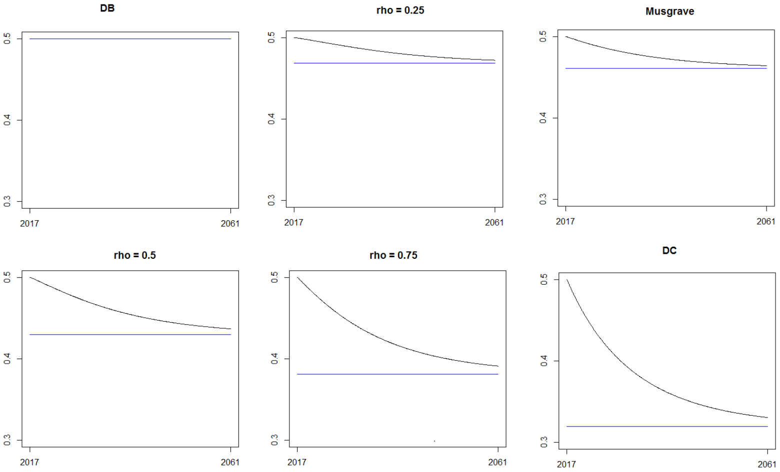
Risk sharing under stochastic optimal control

Contribution rate



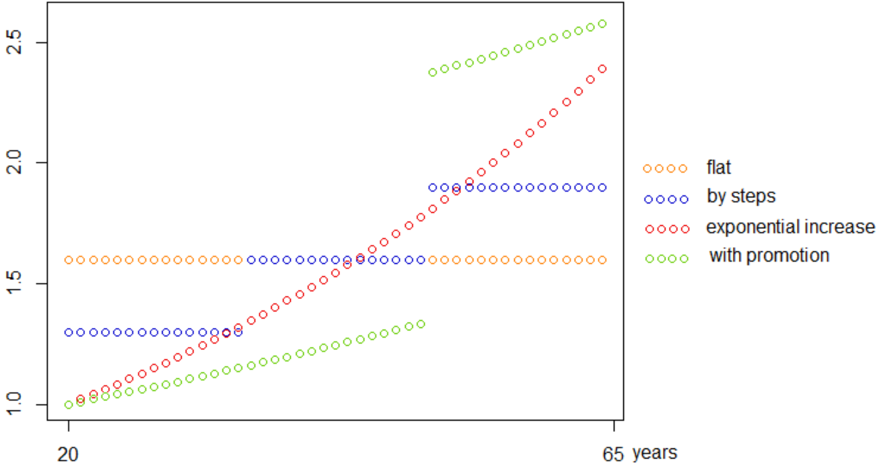
Risk sharing under stochastic optimal control

Replacement rate



Salaries

4 career profiles



Another risk sharing

Remember, the second risk sharing model is

$$\gamma\, DB + (1-\gamma)\, DC\ .$$

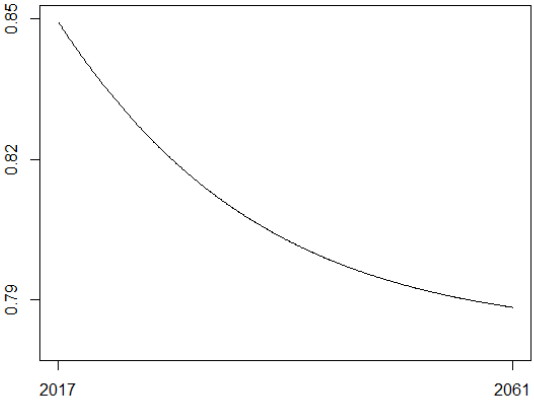
We simulate scenarios with

$\gamma = \{0, 0.25, 0.5, 0.75, 1\}$ and the Musgrave rule with $\tilde{\gamma}_t$.

Another risk sharing

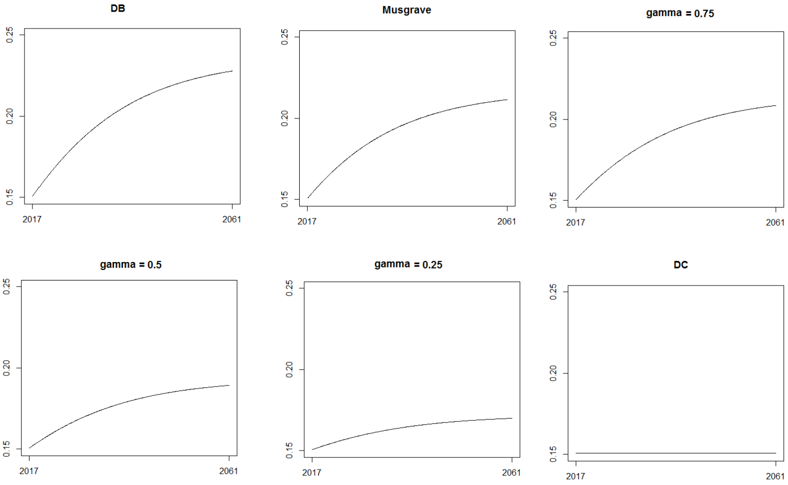
Musgrave rule

$$\tilde{\gamma}_t = \frac{M\,D_t - \pi_0 - \pi_0\,M\,D_t}{(1 + M\,D_t)\,(\delta_0\,D_t - \pi_0)}$$



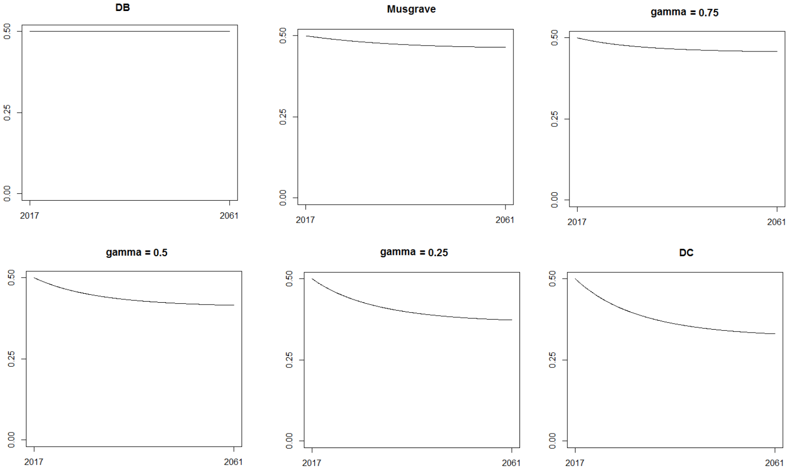
Another risk sharing

Contribution rate



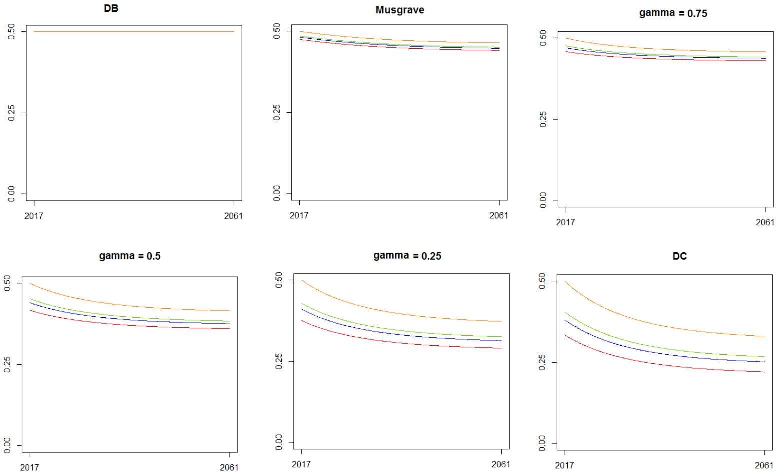
Another risk sharing

Mean replacement rate



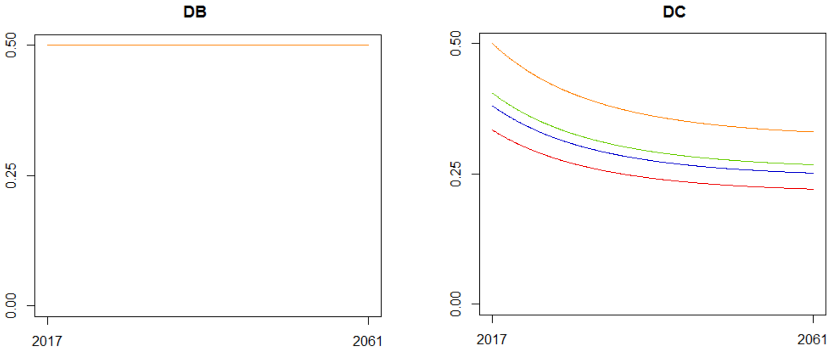
Another risk sharing

Individual replacement rate



Another risk sharing

Individual replacement rate



The replacement rate is

- constant,
- the same for everyone.

Two effects :

- the demographic risk,
- the salary risk.

Conclusions

- Ageing induces an ineluctable and significant increase of the dependence ratio in the coming decades.

- In the aim to maintain a balanced PAYG system, we propose two different **risk sharing** :

  mix between the extreme DB and DC schemes.

Future research

- **Optimal** risk sharing through the processes ρ_t and γ_t .

- Integration of the proposed risk sharing models within the **NDC system** (with a variable contribution rate).

Risk sharing for public pension schemes

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References I

Andrew Cairns.

Some notes on the dynamics and optimal control of stochastic pension fund models in continuous time.

Astin Bulletin, 30(01) :19–55, 2000.

Pierre Devolder and Sébastien de Valeriola.

Pension design and risk sharing : mix solutions between DB and DC for public pension schemes.

Public pension system : The greatest economic challenge for the 21st century, 2017.

Federal Planning Bureau.

Spf economie – direction générale statistique.

2014.

References II

Frédéric Gannon, Florence Legros, and Vincent Touzé.
Automatic adjustment mechanisms and budget balancing of pension schemes.
In *Pension Benefits and Social Security symposium, Lyons*, 2013.

The High Council of Finance.
Committee for research on ageing – annual report.
July 2016.

Monique Jeanblanc, Rose-Anne Dana, et al.
Financial markets in continuous time.
Technical report, Paris Dauphine University, 2003.

Richard A Musgrave.
A Reappraisal of Financing Social Security.
Social Security Financing, 89 :103–122, 1981.

References III

Nizar Touzi.

Stochastic control problems, viscosity solutions, and application to finance.

Technical report, CREST, Paris, 2002.

Carlos Vidal-Meliá, María del Carmen Boado-Penas, and Ole Settergren.

Automatic balance mechanisms in pay-as-you-go pension systems.

The Geneva Papers on Risk and Insurance Issues and Practice, 34(2) :287–317, 2009.