Journal of Logic, Language and Information (2006) 15: 1–3
DOI: 10.1007/s10849-005-9003-9

Introduction

There is a clear connection between probability and logic: both appear to tell us how we should reason. But how, exactly, are the two concepts related?

The need for a coherent answer to this question has become increasingly urgent in the past few years, particularly in the field of artificial intelligence. There, both logical and probabilistic techniques are routinely applied in an attempt to solve complex problems such as parsing natural language and determining the way proteins fold. The hope is that some combination of logic and probability will produce better solutions. After all, both natural language and protein molecules have some structure that admits logical representation and reasoning; yet inherent uncertainties also demand the use of probabilistic methods: this structure is only partially known and does not in any case fully determine a solution – context or environment also play a role.

Objective Bayesianism offers one answer to this question of the relationship between probability and logic. Objective Bayesians argue that an agent's initial degrees of belief (her *prior* belief distribution) should be consistent with her background knowledge but should be non-committal in other respects – i.e., her degrees of belief should be far from the extremes of 0 and 1 unless such strong commitment is warranted by background knowledge. Probability theory is then used to draw conclusions from a prior assignment of beliefs. According to objective Bayesianism, probability generalises deductive logic: deductive logic tells us which conclusions are certain, given a set of premises, while probability tells us the extent to which one should believe a conclusion, given the knowledge of the premises (certain conclusions being awarded full degree of belief). Typically moreover, the premises objectively (i.e. uniquely) determine the degree to which one should believe a conclusion.[1]

The papers in this volume address objective Bayesianism and other proposals for combining probability and logic. The papers all stem from presentations made to the Second Workshop on Combining Probability and Logic (Progic 2005), an interdisciplinary workshop held at the London School of Economics on 6th–8th July 2005.

The objective Bayesian approach is currently widely applied in statistics. However, Bayesian statisticians are rarely explicitly objectivist. More often they tacitly

[1] Williamson (2005a, Chapters 2 and 5) provides an introduction to interpretations of probability including the objective Bayesian interpretation. Williamson (2005c) discusses some of the challenges that face objective Bayesianism.

follow an objectivist methodology by using non-committal or non-informative priors, as opposed to following the subjectivist approach of eliciting degrees of belief from human agents. One difficulty facing the objective Bayesian approach in statistics is that it advocates the use of non-informative belief distributions that are not strictly probability functions – so-called *improper priors*; these can lead to computational and conceptual problems. In their paper, Jukka Corander and Pekka Marttinen propose a new technique for model learning, based on the *Bayesian entropy criterion* (BEC), that sets out to avoid the difficulties associated with improper priors.

While non-committal priors offer one mechanism for achieving objectivity of inference, *interval-valued probabilities* offer another.[2] Assuming that background knowledge constrains a degree of belief to lie within an interval, there are two ways of proceeding. The objective Bayesian approach advocates choosing the most non-committal point within the interval as one's degree of belief. On the other hand the interval-valued approach advocates treating the *whole interval* as one's partial belief. As with improper priors, intervals are not strictly-speaking probabilities, but intervals do not seem to lead to the same kind of problems when it comes to model selection. Frank Coolen shows how interval-valued probabilities can be applied to the task of non-parametric predictive inference, and shows how the ensuing approach compares with the objective Bayesian methodology. Gregory Wheeler is concerned with the minimum point of such an interval – the lower bound on the probability of a proposition. Wheeler argues that these lower bounds can be used to underpin a logic of rational acceptance and to shed light on the lottery paradox and the paradox of the preface.

Richard Bradley's paper opens with a paradox that motivates a well-known triviality theorem which in turn suggests that the probability of a conditional cannot be a conditional probability. Bradley considers Popper-Miller probability (which also differs from the standard notion of probability), and shows that even if this notion of probability is weakened by dropping a monotonicity axiom, one can still derive a triviality result. Hence Popper-Miller probability fails to offer hope for a simple account of material conditionals in a probabilistic logic.

While one needs to provide some account of how to treat the material conditional in a probabilistic logic, one also needs to be able to cope with causal conditionals, such as means-end relations. Jesse Hughes, Albert Esterline and Bahram Kimiaghalam argue that *propositional dynamic logic* (PDL) can be used to provide semantics for means-end relations, and that adding probabilities and fuzzy predicates to PDL allows one to measure the efficacy of means-end relations.

Kathryn Blackmond Laskey proposes a logic for computing that explicitly models the way an agent should learn from experience as well as indeterministic processes that give rise to that experience. This leads to a combination of

[2] (Williamson (2005b).

first-order logic, Bayesian reasoning and quantum physics that is based on Laskey's multi-entity Bayesian networks (MEBNs). MEBNs can be used both to reason with causal conditionals and to model quantum systems.

A probabilistic logic integrates logic and probability into a new formalism, with new semantics and new methods of inference. But there is another way of combining probability and logic: probabilistic and logical formalisms can be kept distinct, but they may interact in fruitful ways. Matt Williams and I pursue this second type of approach. In our paper, interactions between a logical formalism – argumentation frameworks – and a distinct probabilistic formalism – causally-interpreted Bayesian nets – are applied to the problem of determining a prognosis for breast cancer.

We see then that there are a plethora of combinations of probability and logic, and that these approaches are being investigated in some detail. Combining probability and logic (progic) is thus now a mature interdisciplinary research topic. It has a hefty agenda: to further develop the particular approaches and their applications, to forge connections between approaches, to compare them and to evaluate them.

Acknowledgements

I am very grateful to the British Society for the Philosophy of Science and to the Centre for the Philosophy of the Natural and Social Sciences at the London School of Economics for supporting Progic 2005. Thanks too to all the speakers and participants at the workshop for their high quality submissions and lively discussion.

References

Williamson, J., 2005a, *Bayesian Nets and Causality: Philosophical and Computational Foundations*, Oxford: Oxford University Press.

Williamson, J., 2005b, "Motivating objective Bayesianism: from empirical constraints to objective probabilities," in *Probability and Inference: Essays in Honour of Henry E. Kyburg Jr.*, W.L. Harper, and G.R. Wheeler (eds.), London: King's College Publications.

Williamson, J., 2005c, "Philosophies of probability: objective Bayesianism and its challenges," in: A. Irvine (ed.), *Handbook of the Philosophy of Mathematics*, Elsevier. Handbook of the Philosophy of Science volume 9.

Jon Williamson
Guest Editor

Department of Philosophy
University of Kent
E-mail: j.williamson@kent.ac.uk