Classical Inductive Logic

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1 From Deductive to Inductive Logic

Consider the following argument in propositional logic:

$$\frac{\begin{array}{r} a \rightarrow b \\ b \end{array}}{a}$$

We can ask whether the argument is deductively valid:

$$a \rightarrow b, b \models a?$$

We know the argument is invalid by considering its truth table:

a	b	$a \rightarrow b$	b	a
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Hence this argument, *affirming the consequent*, is deemed fallacious: $a \rightarrow b, b \not\models a$.

While the argument seems a poor one from a deductive point of view, we can ask:

Partial Entailment. To what extent is the conclusion *plausible*, given the premisses?

IE What level Y of plausibility attaches to the conclusion, given the premisses?

$$a \rightarrow b, b \approx\!\!\!\mid a^Y.$$

Support. To what extent do the premisses make the conclusion *more* plausible than it is in their absence?

IE To what extent do the premisses *support* the conclusion?

EG If $a \rightarrow b, b \mid\!\approx a^y$ and $\mid\!\approx a^z$, then degree of support $= y - z$.

Classical inductive logic. Degree of partial entailment is the proportion of those truth assignments that make the premisses true that also make the conclusion true (Wittgenstein, 1922, §5.15).

NB That this proportion can be read off a standard truth table was noted by Wittgenstein (1922, §5.151).

a	b	$a \rightarrow b$	b	a
T	T	[T]	[T]	[T]
T	F	F	F	T
F	T	[T]	[T]	F
F	F	T	F	F

$$a \rightarrow b, b \models\!\!\approx a^{1/2}.$$

Classical inductive logic can be thought of in terms of three principles:

CIL1. Measure inductive plausibility by probability.

We can augment each line of a truth table with the probability that the atomic propositions take the truth values specified on that line:

P	a	b	$a \rightarrow b$	b	a
x_1	T	T	T	T	T
x_2	T	F	F	F	T
x_3	F	T	T	T	F
x_4	F	F	T	F	F

CIL2. These probabilities should fit the premisses.

EG States where one or more premisses turn out false should have zero probability.

P	a	b	$a \rightarrow b$	b	a
x_1	T	T	T	T	T
0	T	F	F	F	T
x_3	F	T	T	T	F
0	F	F	T	F	F

CIL3. If the premisses fail to distinguish between two possible truth assignments, then they are equally plausible.

P	a	b	$a \rightarrow b$	b	a
$\frac{1}{2}$	T	T	T	T	T
0	T	F	F	F	T
$\frac{1}{2}$	F	T	T	T	F
0	F	F	T	F	F

We find then that probability $\frac{1}{2}$ attaches to the conclusion:

P	a	b	$a \rightarrow b$	b	a
$\boxed{\frac{1}{2}}$	T	T	T	T	$\boxed{\text{T}}$
0	T	F	F	F	T
$\frac{1}{2}$	F	T	T	T	F
0	F	F	T	F	F

So,

$$a \rightarrow b, b \mathrel{|\!\approx} a^{1/2}.$$

Classical inductive logic follows naturally from the classical interpretation of probability:

> The theory of chance consists in reducing all the events of the same kind to a certain number of cases equally possible, that is to say, to such as we may be equally undecided about in regard to their existence, and in determining the number of cases favorable to the event whose probability is sought. The ratio of this number to that of all the cases possible is the measure of this probability, which is thus simply a fraction whose numerator is the number of favorable cases and whose denominator is the number of all the cases possible. (Laplace, 1814, pp. 6–7.)

In the absence of the premisses we have the following probability table:

P	a	b	a
$\boxed{\frac{1}{4}}$	T	T	$\boxed{\text{T}}$
$\boxed{\frac{1}{4}}$	T	F	$\boxed{\text{T}}$
$\frac{1}{4}$	F	T	F
$\frac{1}{4}$	F	F	F

Hence,

$$\approx\!\!\!\!| \; a^{1/2}.$$

$\therefore$ The degree to which the premisses support the conclusion is $1/2 - 1/2 = 0$.

EG Consider the following argument:

$$\begin{array}{r} (a \wedge b) \rightarrow c \\ a \\ \hline c \end{array}$$

a	b	c	$(a \wedge b) \rightarrow c$	a	c
T	T	T	$\boxed{\text{T}}$	$\boxed{\text{T}}$	$\boxed{\text{T}}$
T	T	F	F	T	F
T	F	T	$\boxed{\text{T}}$	$\boxed{\text{T}}$	$\boxed{\text{T}}$
T	F	F	$\boxed{\text{T}}$	$\boxed{\text{T}}$	F
F	T	T	T	F	T
F	T	F	T	F	F
F	F	T	T	F	T
F	F	F	T	F	F

- This is an invalid argument (line 4).
- $(a \wedge b) \rightarrow c, a \models\!\approx c^{2/3}$ (lines 1,3,4).
- $\models\!\approx c^{1/2}$ (lines 1-8).

∴ The premisses support the conclusion to degree $2/3 - 1/2 = 1/6$.

Consider again the question $a \to b, b \mathrel{|\!\!\approx} a^?$, but in the context of the following argument:

$$\frac{\begin{array}{r}\text{If the month contains the letter `}r\text{' then the oyster is not toxic}\\ \text{The oyster is not toxic}\end{array}}{\text{The month contains the letter `}r\text{'}}$$

It might be argued that the context makes the conclusion more plausible:

- ∘ There is some implicit knowledge, namely that the probability of a is, in the absence of other information, $8/12 = 2/3$.
- ∴ The inference should be evaluated relative to the background $\Gamma = \{a^{2/3}\}$.

P	a	b	$a \to b$	b	a
$\boxed{\frac{2}{3}}$	T	T	T	T	T
0	T	F	F	F	T
$\boxed{\frac{1}{3}}$	F	T	T	T	F
0	F	F	T	F	F

Then $\Gamma; a \to b, b \mathrel{|\!\!\approx} a^{2/3}$.

NB Degree of support remains 0 since $\Gamma; \mathrel{|\!\!\approx} a^{2/3}$.

Suppose that a test of the oyster determines toxicity to within a specified margin of error:

$$\Gamma; a \rightarrow b, b^{[.9,1]} \mathrel{|\!\!\approx} a^{?}$$

Now:

- $a \rightarrow b$ forces $x_2 = 0$.
- Γ implies that $x_1 = 2/3$.
- $b^{[.9,1]}$ implies $x_1 + x_3 \in [.9, 1]$.

∴ $x_4 \in [0, .1]$ and $x_3 = 1/3 - x_4$.

CIL3 motivates setting x_3 and x_4 as equal as possible.

IE $x_3 = 7/30$ and $x_4 = 1/10$.

P	a	b	$a \rightarrow b$	b	a
2/3	T	T	T	T	T
0	T	F	F	F	T
7/30	F	T	T	T	F
1/10	F	F	T	F	F

$$\Gamma; a \rightarrow b, b^{[.9,1]} \mathrel{|\!\!\approx} a^{2/3}$$

2 Learning from experience

Carnap (1945, p. 81): classical inductive logic fails to allow learning from experience:

$$\approx\!\!\!\mid\; Br_{101}^{1/2}$$

but also

$$Br_1, \ldots, Br_{100} \approx\!\!\!\mid\; Br_{101}^{1/2}$$

since $Br_1, \ldots, Br_{100}$ are logically independent of Br_{101}.

IE This represents an inability to learn from experience.

This observation had previously been made by George Boole.

Wittgenstein pointed out that this phenomenon is due to the propositions being logically independent.

IE having no propositional variables in common, when viewed from the perspective of propositional logic.

An inductive logic needs to capture:

Inductive Entailment. The degree to which an observed sample of ravens makes plausible the proposition that the next raven is black.

- An ampliative concept that can link logically independent propositions.
- CIL fails to capture this concept.
- Carnap (1945) abandoned the classical notion of partial entailment to try to capture learning from experience.

Logical Entailment. The degree to which $A \vee B$ makes plausible proposition A.

- A non-ampliative concept, attributable to logical dependence.
- CIL is required to capture this concept (Wittgenstein, Kemeny and Oppenheim).
- Kemeny and Oppenheim (1952) focused on classical partial entailment to the exclusion of learning from experience.

Salmon argued that it is not possible to capture both phenomena in a single inductive logic:

> if degree of confirmation is to be identified with partial entailment, then $c^{\dagger}$ [i.e., CIL] is the proper confirmation function after all, for it yields the result that *p* is probabilistically irrelevant to *q* whenever *p* and *q* are completely independent and there is no partial entailment between them. . . . Unfortunately for induction, statements strictly about the future (unobserved) are completely independent of statements strictly about the past (observed). Not only are they deductively independent of each other, but also they fail to exhibit any partial entailment. The force of Hume's insight that the future is logically independent of the past is very great indeed. It rules out both full entailment and partial [i.e., logical] entailment. If partial [= logical] entailment were the fundamental concept of inductive logic, then it would in fact be impossible to learn from experience. (Salmon, 1967, pp. 731–2.)

3 Objective Bayesian epistemology

An agent with evidence E and language $\mathcal{L}$ should apportion the strengths of her beliefs according to three norms:

Probability. Her belief function should be a probability function.

✓ Dutch book considerations: minimise worst-case expected loss.

Calibration. Her belief function should fit her evidence.

EG Her degrees of belief should be set to frequencies where known.

✓ Long-run betting: minimise worst-case expected loss.

Equivocation. Her belief function should equivocate sufficiently between basic possibilities.

✓ Scoring rules: minimise worst-case expected loss.

Probability. Her belief function P_E should be a probability function, $P_E \in \mathbb{P}$.

- **P1.** $P(\omega) \geq 0$ for each $\omega \in \Omega_n$,
- **P2.** $P(\tau) = 1$ for some tautology $\tau \in S\mathcal{L}$, and
- **P3.** $P(\theta) = \sum_{\omega \models \theta} P(\omega)$ for each $\theta \in S\mathcal{L}$.

Calibration. Her belief function should be compatible with her evidence, $P_E \in \mathbb{E} \subseteq \mathbb{P}$.

- **C.** $\mathbb{E} = \langle \mathbb{P}^* \rangle \cap \mathbb{S}$.

Equivocation. Her belief function should equivocate sufficiently between basic possibilities, $P_E \in \downarrow\mathbb{E} \subseteq \mathbb{E}$.

- **E.** $\downarrow\mathbb{E} = \{P \in \mathbb{E} : d(P, P_=) \text{ is minimised}\}$.
  - KL-divergence $d(P, P_=) = \sum_\omega P(\omega) \log P(\omega)/P_=(\omega)$.

∴ Maximum Entropy Principle (Jaynes, 1957): choose $P_E \in \mathbb{E}$ with maximum entropy

$$H(P) = -\sum_\omega P(\omega) \log P(\omega).$$

NB No updating rule required: if E changes to E' then P_E changes to $P_{E'}$.

(Williamson, J. (2010). *In defence of objective Bayesianism*. Oxford University Press, Oxford.)

Objective Bayesian inductive logic

- $\Gamma; \varphi^{X_1}, \ldots, \varphi_k^{X_k} \mid\!\approx^{\circ} \psi^Y$ iff $P_{\Gamma \cup \{\varphi^{X_1}, \ldots, \varphi_k^{X_k}\}}(\psi) \in Y$, for any P satisfying the norms of OBE.

OBIL preserves classical inductive logic:

- CIL1 follows from the Probability norm.
- CIL2 follows from the Calibration norm.
- CIL3 follows from the Equivocation norm.

∴ OBIL captures logical entailment.

Learning from experience is possible

EG Observing ravens $r_1, \ldots, r_{101}$ to see if they are black B.

- $P_\varnothing(Br_{101}) = 1/2 = P_\varnothing(Br_{101} \mid Br_1 \wedge \cdots \wedge Br_{100})$

NB We need $P_{\Gamma \cup \{Br_1 \wedge \cdots \wedge Br_{100}\}}(Br_{101})$.

- Suppose the agent grants that $Br_1, \ldots, Br_{100}$ and that outcomes are iid.
- Let $1-\epsilon$ be the minimum degree to which she would need to believe $P_R^*(B) \geq x$ for her to grant it.
- Frequentist statistics can determine δ such that $P_S^*(|f_S - P_R^*(B)| \leq \delta) = 1-\epsilon$.
- Calibration: $P_E(P_R^*(B) \geq 1-\delta) = 1-\epsilon$.
- The agent grants that $P_R^*(B) \geq 1-\delta$.
- Calibration: $P_{E'}(Br_{101}) \geq 1-\delta$.
- Equivocation: $P_{E'}(Br_{101}) = 1-\delta$.

$\therefore$ $\Gamma; Br_1 \wedge \cdots \wedge Br_{100} \mathrel{|\!\!\!\approx^{\circ}} Br_{101}^{1-\delta}$.

IE Learning from experience is possible.

NB Statistical theory is playing a crucial role here.

- Inductive entailment is captured by statistical theory and Calibration.
- Logical entailment is captured by Equivocation.

Summary and Links

- Partial entailment is an overloaded relation:
  - Inductive entailment can’t be captured by classical inductive logic.
  - Logical entailment can be capured by CIL.
- Statistical theory is best equipped to capture inductive entailment.
  - Objective Bayesian inductive logic can capture both forms of entailment.

Paper. From Bayesian epistemology to inductive logic, Journal of Applied Logic, to appear.

- `http://www.kent.ac.uk/secl/philosophy/jw/2011/BEIL.pdf`

✓ OBIL extends naturally to predicate languages.

✓ Responses to standard criticisms of inductive logic.

Project. From objective Bayesian epistemology to inductive logic, AHRC 2012–15.

- `http://www.kent.ac.uk/secl/philosophy/jw/2012/fobetil/`
- Visitors welcome!

Predicate Languages

- $\mathcal{L}$ is a first-order predicate language without equality,
  - with finitely many predicate symbols,
  - with a constant symbol $t_1, t_2, \ldots$, for each element of the domain.
- For $n \geq 1$, let $\mathcal{L}_n$ be the finite predicate language involving only constants $t_1, \ldots, t_n$.
- Let $A_1, \ldots, A_{r_n}$ be the atomic propositions expressible in $\mathcal{L}_n$.
- An *atomic n-state* ω_n is an atomic state $\pm A_1 \wedge \cdots \wedge \pm A_{r_n}$ of $\mathcal{L}_n$.

Probability. Degrees of belief should satisfy the axioms of probability.

PP1. $P(\omega_n) \geq 0$ for each $\omega_n \in \Omega_n$ and each n,

PP2. $P(\tau) = 1$ for some tautology $\tau \in S\mathcal{L}$,

PP3. $P(\theta) = \sum_{\omega_n \models \theta} P(\omega_n)$ for each quantifier-free proposition θ, for any n large enough that $\mathcal{L}_n$ contains all the atomic propositions occurring in θ, and

PP4. $P(\exists x \theta(x)) = \sup_m P\left(\bigvee_{i=1}^{m} \theta(t_i)\right)$.

NB A probability function is determined by its values on the atomic n-states.

Calibration. As before, $P_E \in \mathbb{E} = \langle \mathbb{P}^* \rangle \cap \mathbb{S}$.

Equivocation. Degrees of belief should be equivocate sufficiently between the basic possibilities that one can express.

- Equivocator $P_{=}(\omega_n) = |\Omega_n|$ for all n and ω_n.
- *n-divergence* $d_n(P, Q) = \sum_{\omega_n \in \Omega_n} P(\omega_n) \log P(\omega_n)/Q(\omega_n)$.
- P is *closer* to R than Q if there is some N such that for all $n \geq N$, $d_n(P, R) < d_n(Q, R)$.

E. P should be sufficiently close to $P_{=}$.

Language Dependence

? To what extent do inferences in this inductive logic depend on the underlying language?

If one can formulate an inference in more than one language then the two formulations will agree as to whether the premisses entail the conclusion.

Theorem 1. *Given predicate languages* $\mathcal{L}^1$ *and* $\mathcal{L}^2$*, suppose that* $\varphi_1, \ldots, \varphi_n, \psi$ *are propositions of both* $\mathcal{L}^1$ *and* $\mathcal{L}^2$*. Let* $\approx\!\!\!\!\!\!|\,^{1}$ *be the entailment relation with respect to* $\mathcal{L}^1$ *and* $\approx\!\!\!\!\!\!|\,^{2}$ *be the entailment relation with respect to* $\mathcal{L}^2$*. Then* $\varphi_1^{X_1}, \ldots, \varphi_k^{X_k} \approx\!\!\!\!\!\!|\,^{1} \psi^Y$ *if and only if* $\varphi_1^{X_1}, \ldots, \varphi_k^{X_k} \approx\!\!\!\!\!\!|\,^{2} \psi^Y$*.*

∴ There is normally no need to spell out the underlying language.

However, there is another sense in which inferences *do* depend on the underlying language:

EG Suppose

- $\mathcal{L}^1$ has two unary predicates, *Green* and *Blue*.
- $\mathcal{L}^2$ just has one unary predicate, *Grue*, which is synonymous with *Green or Blue*.

Then we have that

$$\mathrel{|\!\!\approx}^1 Green(t_1) \vee Blue(t_1)^{3/4}$$

but

$$\mathrel{|\!\!\approx}^2 Grue(t_1)^{1/2}.$$

? Which is the correct inference?

The Bayesian would say that both are correct:

- If your language were $\mathcal{L}^1$ then you ought to believe that t_1 is Green or Blue to degree 3/4.
- If your language were $\mathcal{L}^2$ you ought to believe that t_1 is Grue to degree 1/2.

✓ Just as degrees of belief should depend on explicit evidence because that evidence tells us about the world, so too should degrees of belief depend on language because language tells us about the world.

  - Evidence tells us facts about the world.
    - Two evidence bases can be compared wrt strength and accuracy.
  - Language tells us about how the world can be carved up.
    - Two languages can be compared wrt how well they carve up the world.

But the Bayesian would also say that both inferences are lacking:

- There is an important piece of information that has not been taken into account:
    - That Grue is synonymous with Green or Blue.

∴ Need a language $\mathcal{L}^3$ in which one can express the proposition

$$\forall x, Grue(x) \longleftrightarrow (Green(x) \vee Blue(x)).$$

Then one can formulate the inference

$$\forall x, Grue(x) \longleftrightarrow (Green(x) \vee Blue(x)) |\!\!\approx^{\circ 3} Grue(t_1)^{3/4}.$$

But we also find that

$$\forall x, Grue(x) \longleftrightarrow (Green(x) \vee Blue(x)) |\!\!\approx^{\circ 3} Green(t_1) \vee Blue(t_1)^{3/4},$$

so there is no inconsistency.

? Is this sort of dependence on language problematic?

Arguably not, given this triviality result:

- A *synonymy map* between predicate languages $\mathcal{L}$ and $\mathcal{L}'$ is a consistent, countable set of propositions of the form $\theta_i \longleftrightarrow \theta_i'$ where the θ_i are propositions of $\mathcal{L}$ and the θ_i' are propositions of $\mathcal{L}'$.

Theorem 2. *If an entailment relation* $\approx\!\!\!|$ *of probabilistic logic with underlying predicate language* $\mathcal{L}$ *is invariant under all synonymy maps between* $\mathcal{L}$ *and* $\mathcal{L}'$*, for all* $\mathcal{L}'$*, then,*

1. $\varphi_1^{X_1}, \ldots, \varphi_k^{X_k} \mathrel{|\!\!\approx} \psi^Y$ *if and only if* $\varphi_1^{X_1}, \ldots, \varphi_k^{X_k} \mathrel{|\!\!\approx} \neg\psi^Y$*,*
2. $\varphi_1^{X_1}, \ldots, \varphi_k^{X_k} \mathrel{|\!\!\approx} \psi^Y$ *implies that* $\{0, 1\} \subseteq Y$*.*

In sum,

- Theorem 1 shows that inferences are independent of the underlying language.
- However, they are not invariant under arbitrary synonymy maps.
- Theorem 2 shows that one cannot demand this stronger invariance condition without trivialising inductive logic.

The Principle of Indifference

? Does this logic fall to paradoxes of the Principle of Indifference.

> The principle of indifference asserts that if there is no *known* reason for predicating of our subject one rather than another of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an *equal* probability. These *equal* probabilities must be assigned to each of several arguments, if there is an absence of positive ground for assigning *unequal* ones. (Keynes, 1921, p. 45)

Keynes restricted the Principle to *indivisible* alternatives:

> In short, the principle of indifference is not applicable to a pair of alternatives, if we know that either of them is capable of being further split up into a pair of possible but incompatible alternatives of the same form as the original pair. (Keynes, 1921, p. 66)

- On a propositional language, the set Ω_n of atomic states represents the partition of indivisible alternatives.
- On a predicate language there is no partition of indivisible alternatives:
    - For every set Ω_n of atomic n-states, there is another set Ω_i for $i > n$ that splits up the original alternatives.

But one can formulate the Principle of Indifference in a way that applies equally to the infinite and the finite case:

POI. If atomic n-states ω_n^* and $\omega_n^\dagger$ are treated symmetrically by the premisses then,

$$\varphi_1^{X_1}, \ldots, \varphi_k^{X_k} \approx\!\!\!\models {\omega_n^*}^Y \text{ iff } \varphi_1^{X_1}, \ldots, \varphi_k^{X_k} \approx\!\!\!\models {\omega_n^\dagger}^Y.$$

Here we can say that ω_n^* and $\omega_n^\dagger$ are *treated symmetrically by the premisses* just in case:

- for any probability function P satisfying the premisses, there is another function satisfying the premisses which swaps the probabilities of ω_n^* and $\omega_n^\dagger$ but which otherwise agrees with P as far as possible.

Interestingly:

Theorem 3. *The Bayesian entailment relation* $\models\!\!\!\approx$ *satisfies POI.*

- **?** Is satisfying the Principle of Indifference a problem?
- ✓ There is no inconsistency: probabilities are attached to a language $\mathcal{L}$ in a consistent way.
- × But paradoxes might still arise when one changes the conceptualisation of a particular problem to an equivalent but different conceptualisation in which the partition of indivisible alternatives is different.
  - IE If one changes the language and at the same time assert an equivalence between certain propositions of the new language and of the old.
    - IE By introducing a synonymy map.
    - EG The *Grue* example above.
    - ✓ But Bayesians can argue that dbs should display this behaviour.
    - ✓ Any demand that inductive logic should be immune to this sort of behaviour is untenable since there is no non-trivial inductive logic that satisfies such a demand (Theorem 2).

In sum, POI holds, but arguably not with problematic consequences.

Universal Hypotheses

POI also implies that many universal hypotheses are given zero probability,

$$\varphi_1^{X_1}, \ldots, \varphi_k^{X_k} \mid\!\approx \forall x \theta(x)^0.$$

× This can be counterintuitive.

EG Finding the first 100 observed ravens to be black offers no support to the conclusion that all ravens are black:

$$Br_1{}^1, \ldots, Br_{100}{}^1, Br_{101}{}^{[1-\delta_0,1]}, Br_{102}{}^{[1-\delta_0,1]}, \ldots \mid\!\overset{\circ}{\approx} \forall x Bx^0.$$

- In general, *no generalisations in, no generalisations out*:
  - If premisses are to raise the probability of a universally quantified proposition away from zero, then those premisses must themselves involve quantifiers:

Theorem 4. *Suppose that, for* $\theta(x)$ *quantifier-free,* $\mid\!\overset{\circ}{\approx} \forall x \theta(x)^0$ *but* $\varphi_1^{X_1}, \ldots, \varphi_k^{X_k} \mid\!\overset{\circ}{\approx} \forall x \theta(x)^Y$ *where* $\inf Y > 0$*. Then* $\varphi_1, \ldots, \varphi_k$ *are not all quantifier-free.*

? Isn’t this a problem for applications to the sciences, which appear to routinely invoke universal generalisations?

Perhaps not:

- Arguably, universal generalisations are not so central to science.
  - In Carnap’s time, largely under the influence of the logical empiricists, scientific theories were understood as collections of universal generalisations.
    - Augmented by statements specifying boundary conditions, bridge laws etc.
  - ∴ Inductive logics that gave universal generalisations probability zero were taken to be refuted by scientific practice.
  - But in the 1980s and 1990s this view of laws was found to be untenable.
    - The ubiquity of ceteris paribus laws and pragmatic laws became recognised.
  - More recently, the Hempelian DN account of explanation, which saw scientific explanations as deductions from universal generalisations, has been replaced by a mechanistic view of explanation.
  - Science is increasingly understood as a body of mechanisms, not generalisations.
  - ∴ The relevance of Theorem 4 to science is less obvious now than it would have appeared a few decades ago.

Also, there are a variety of attitudes one can take towards universal generalisations.

- Bayesian epistemology distinguishes what is *believed* and what is *granted*.
- Given what is already granted, BE provides rational norms for degrees of belief:
  - In terms of inductive logic, it tells us how strongly one should believe a conclusion proposition having granted some premisses.
- Moreover, different norms cover granting and believing.
  - Grounds for *granting* include coherence, simplicity, strength, accuracy, technical convenience, unifying power etc.
  - But propositions should only be *believed* to the extent warranted by their Bayesian probability relative to what is granted.
- So a generalisation may have probability zero yet be ripe for granting.

∴ The Bayesian can argue that:
  - One should remain sceptical about universal hypotheses that have probability 0.
  - Yet one can go on to grant those same hypotheses for other reasons.

In sum,

- Universal generalisations appear to play less of a role in science than previously thought.
- The key question may be whether they should be *granted*, rather than believed.

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