Possible Semantics for a Common Framework of Probabilistic Logics

Rolf Haenni[1], Jan-Willem Romeijn[2], Gregory Wheeler[3], and Jon Williamson[4]

[1] Bern University of Applied Sciences, Engineering and Information Technology, 2501 Biel, Switzerland
rolf.haenni@bfh.ch

[2] University of Groningen, Faculty of Philosophy, 9712 GL Groningen, The Netherlands
J.W.Romeijn@rug.nl

[3] Universidade Nova de Lisboa, Department of Computer Science, 2829-516 Caparica, Portugal
grw@fct.unl.pt

[4] University of Kent, SECL, Philosophy, Canterbury, CT2 7NF, United Kingdom
j.williamson@kent.ac.uk

Summary. This paper proposes a common framework for various probabilistic logics. It consists of a set of uncertain premises with probabilities attached to them. This raises the question of the strength of a conclusion, but without imposing a particular semantics, no general solution is possible. The paper discusses several possible semantics by looking at it from the perspective of probabilistic argumentation.

1 Introduction

If the premises of a valid logical inference are not entirely certain, how certain is its conclusion? To find an answer to this is an important question, it is necessary to overcome the restrictions and limits of the classical fields of logical and probabilistic inference. This simple observations is not entirely new [3, 4, 6, 9, 22, 30, 26], but attempts of building such unifying *probabilistic logics* (or *logics of probability*) are rather sparse, especially in comparison with the long traditions of logic and probability theory as independent disciplines both in philosophy and in science.[1] Nevertheless, probabilistic logic is nowadays a rapidly developing interdisciplinary research topic with contributions from philosophical logic [1, 8, 17, 19, 31] and Artificial Intelligence [7, 10, 18, 25, 24, 27], but also from mathematics, linguistics, statistics, and decision theory [2, 20]. While it is clear that logic and probability theory are intimately related, the exact shape of this relationship is still the subject of an ongoing debate.

In principle, there are at least two different ways of constructing a combined theory of logical and probabilistic inference, depending on whether logic or

[1] For more information about the historical account of probabilistic logics, we refer to the excellent survey in [17].

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probability theory is at its center. The majority of approaches in the literature is logic-centered, either by defining a probability function on the sentences of the logic [9, 30, 25] or by incorporating probabilities into the syntax of the logic [7, 24]. In the theory of probabilistic argumentation [11, 15, 21], where the available knowledge is partly encoded as a set of logical premises and partly as a fully specified probability space, the starting point is neither biased towards logic, nor is it biased towards probability. This setting gets particularly interesting when some of the logical premises include variables that are not contained in the probability space. The two classical questions of the probability and the logical deducibility of a hypothesis can then be replaced by the more general question of the probability of a hypothesis being logically deducible from the premises.

In Section 2, we first propose a neutral common framework for a variety of different probabilistic logics. The framework as such has no particular semantics, but we will shortly discuss what most people would probably consider its "standard semantics". In Section 3, we first give a short summary of the theory of probabilistic argumentation, which then allows us to discuss various semantics for the common framework. Hence, the goal of this paper is to establish a link between probabilistic argumentation and other probabilistic logics via the common framework.

2 Probabilistic Logics

The principal goal of any *probabilistic logic* (sometimes called *probability logic* [1, 16, 31], or *progic* for short) is to combine the capacity of probability theory to handle uncertainty with the capacity of deductive logic to cope with qualitative and structural knowledge such as logical relationships. As most probabilistic logics are constructed on top of an existing logic (propositional logic in the simplest case), probabilities are usually treated as an addendum rather than as an integral part of the theory. In this section, we propose such a simple addendum, in which probabilities (or sets of probabilities) are attached to premises to represent their respective uncertainties. This then raises the question of the extent to which a possible conclusion follows from the uncertain premises. Given the simplicity and generality of the proposed extension, which allows it to be taken as a common unifying umbrella for many existing probabilistic logics, we will refer to as the *progic framework*.

2.1 The Progic Framework

In a classical logic, the fundamental question of interest is whether a conclusion ψ is logically entailed by a given set of premises $\Phi = \{\varphi_1, \ldots, \varphi_n\}$. Logical inference is thus essentially a problem of verifying the entailment relation $\models$ between Φ and ψ. The entailment relation itself is usually defined in terms of a subset relation $\subseteq$ of corresponding sets of truth assignments (models) in the respective logic.

To augment the fundamental question of classical logic towards probabilistic logic, we will now consider a set of premises with probabilities attached to them. In the simplest case, this means that each premise φ_i has an attached probability $x_i \in [0,1]$, but to be as general as possible, we may also allow the case where a set of probabilities $X_i \subseteq [0,1]$ is attached to each premise φ_i. In this augmented setting, which includes the special case of sharp probabilities by $X_i = \{x_i\}$, the traditional question of classical logic turns into a more general question of the form

$$\varphi_1^{X_1}, \ldots, \varphi_n^{X_n} \models \psi^Y, \tag{1}$$

where the set $Y \subseteq [0,1]$ is intended to represent the extent to which the conclusion ψ follows from the premises.[2] This is a very general question, which covers a multitude of frameworks of existing probabilistic logics. We will thus refer to is as the *general progic framework* (or *progic framework* for short). Note that the problem is the determination of the set Y itself, not the verification of the entailment relation for a given Y. Needless to say that the determination of Y is heavily dependent on the semantics imposed by the chosen framework. In the next subsection, we will discuss one of the most straightforward semantics for the progic framework.

2.2 The Standard Semantics

In the so-called *standard semantics* of the progic framework, we consider each attached probability set X_i as a constraint for the probability $P(\varphi_i)$ in a corresponding probability space. For the sake of simplicity, we will restrict the premises to be propositional sentences. Formally, we write $V = \{Y_1, \ldots, Y_r\}$ to denote the set of involved Boolean variables Y_i, each with a set $\Omega_i = \{0,1\}$ of possible values. In the corresponding propositional language $\mathcal{L}_V$, we use propositional symbols y_i as placeholders for $Y_i = 1$. The Cartesian product $\Omega_V = \Omega_1 \times \cdots \times \Omega_r = \{0,1\}^r$ then contains the set of all possible truth assignments of the propositional language, each of which representing a possible (state of the) world. For a given propositional sentence $\varphi \in \mathcal{L}_V$, we write $[\![\varphi]\!] \subseteq \Omega_V$ to denote the set of truth assignments for which φ evaluates to 1 (according to the usual semantics of propositional logic), and we say that φ entails ψ, or that $\varphi \models \psi$ holds, iff $[\![\varphi]\!] \subseteq [\![\psi]\!]$.

To make a connection to probability theory, let Ω_V play the role of a finite sample space. The finiteness of Ω_V allows us to work with the σ-algebra 2^{Ω_V} of all subsets of Ω_V, i.e. we obtain a probability space $(\Omega_V, 2^{\Omega_V}, P)$ for any measure $P : 2^{\Omega_V} \rightarrow [0,1]$ that satisfies the Kolmogorov's probability axioms. With $\mathbb{P}$ we denote the set of all such probability measures for a given set of variable V. Note that we adopt the usual notational convention of writing $P(\varphi)$ rather than $P([\![\varphi]\!])$ for the probability of the event $[\![\varphi]\!]$.

According to the above-mentioned general idea of the standard semantics, we consider each set X_i as a constraint $P(\varphi_i) \in X_i$ for the unknown probability

[2] For $X_i = \{1\}$, this general setting degenerates into the classical problem of logical inference.

measure P. Formally, let $\mathbb{P}_i = \{P \in \mathbb{P} : P(\varphi_i) \in X_i\}$ denote the set of all probability measures satisfying the constraint for the i-th premise. The intersection of all these sets, $\mathbb{P}_* = \mathbb{P}_1 \cap \cdots \cap \mathbb{P}_n$, defines then the set of probability measures satisfying all constraints. From this, we obtain with $Y = \{P(\psi) : P \in \mathbb{P}_*\}$ a simple solution for the generalized inference problem of the progic framework. Note that inference according to the standard semantics can be seen as a generalization of classical logical inference, which is concerned with a continuum of truth assignments in form of all possible probability measures.

An important special case of the above setting arises when the attached probability sets X_i are all functionally unrelated intervals, i.e. $X_i = [\ell_i, u_i]$. This means that all sets $\mathbb{P}_i$ are convex, which implies that $\mathbb{P}_*$ is also convex and that Y is again an interval with a lower and an upper bound.[3] The lower and upper bounds of Y are usually denoted by $\underline{P}(\psi) = \min\{P(\psi) : P \in \mathbb{P}_*\}$ and $\overline{P}(\psi) = \max\{P(\psi) : P \in \mathbb{P}_*\}$, respectively. Note that the convexity of $\mathbb{P}_*$ guarantees that $\underline{P}$ and $\overline{P}$ are among the extremal points of $\mathbb{P}_*$. Interestingly, we may obtain an interval for Y even if all sets X_i are singletons. From a computational point of view, we can translate the problem of finding Y according to the standard semantics into a (very large) linear optimization problem, e.g. with three constraints $P(\varphi_i) \geq \ell_i$, $P(\varphi_i) \leq u_i$, and $P(\varphi_i) = \sum_{\omega \in [\![\varphi_i]\!]} P(\{\omega\})$ for all premises [1, 25].

Example 1. To illustrate the standard semantics, consider two premises $(a \wedge b)^{[0,0.25]}$ and $(a \vee \neg b)^{\{1\}}$. For the specification of a probability measure with respect to the corresponding 2-dimensional sample space $\{0,1\}^2$ at least three parameters are needed (the size of the sample space minus 1). This means that the set of all possible probability measures $\mathbb{P}$ can be nicely depicted by a tetrahedron (3-simplex) with maximal probabilities for the state descriptions $a \wedge b$, $a \wedge \neg b$, $\neg a \wedge b$, and $\neg a \wedge \neg b$ at each of its four extremities. This tetrahedron is depicted in Fig. 1, together with the convex sets $\mathbb{P}_1$, $\mathbb{P}_2$, and $\mathbb{P}_*$. The picture also shows that $Y = [0,1]$ is the result for the conclusion a, whereas $Y = [0, 0.25]$ is the result for the conclusion b.

3 Probabilistic Argumentation

The theory of *probabilistic argumentation* [11, 13, 15, 21] is first of all driven by the general idea of putting forward the pros and cons of a hypothesis in question, from which it derives its name. The weights of the resulting logical arguments and counter-arguments are measured by probabilities, which are then turned into (sub-additive) *degrees of support* and (super-additive) *degrees of possibility*. Intuitively, degrees of support measure the presence of evidence supporting the hypothesis, whereas degrees of possibility measure the absence of evidence refuting the hypothesis. For this, probabilistic argumentation is concerned with probabilities of a particular type of event of the form "*the hypothesis is a logical consequence*" rather than "*the hypothesis is true*", i.e. very much like Ruspini's

[3] Convex set of probability measures are sometimes called *credal sets* [5, 23].

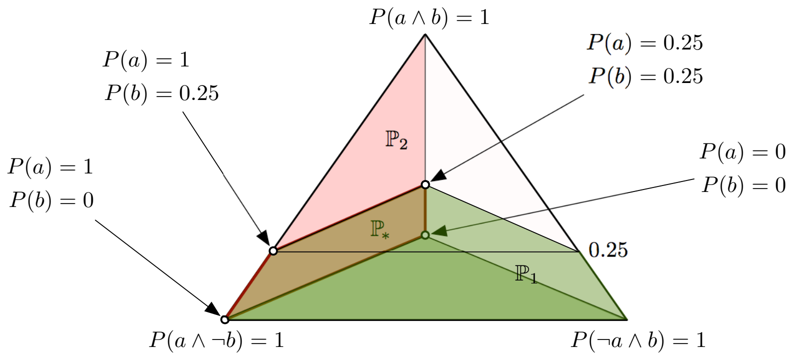

Fig. 1. The set $\mathbb{P}$ of all possible probability measures for the sample space $\{0,1\}^2$, depicted as a tetrahedron, together with the convex sets $\mathbb{P}_1$, $\mathbb{P}_2$, and $\mathbb{P}_*$ of Example 1

epistemic probabilities [28, 29]. Apart from that, they are classical additive probabilities in the sense of Kolmogorov's axioms.

3.1 Degrees of Support and Possibility

Probabilistic argumentation requires the available evidence to be encoded by a finite set $\Phi = \{\varphi_1, \ldots, \varphi_n\} \subset \mathcal{L}_V$ of sentences in a logical language $\mathcal{L}_V$ (over a set of discrete variables V) and a fully specified probability measure $P : 2^{\Theta_W} \rightarrow [0,1]$, where Θ_W denotes the discrete sample space generated by a subset $W \subseteq V$ of so-called *probabilistic variables*. These are the theory's basic ingredients. There are no further assumptions regarding the specification of the probability measure P (we may for example use a Bayesian network) or the language $\mathcal{L}_V$.

Definition 1. *A* probabilistic argumentation system *is a quintuple*

$$\mathcal{A} = (V, \mathcal{L}_V, \Phi, W, P), \tag{2}$$

where V, $\mathcal{L}_V$, Φ, W, and P are as defined above [13].

For a given probabilistic argumentation system $\mathcal{A}$, let another logical sentence $\psi \in \mathcal{L}_V$ represent the hypothesis in question. For the formal definition of degrees of support and possibility, consider the subset of Θ_W, whose elements, if assumed to be true, are each sufficient to make ψ a logical consequence of Φ. Formally, this set of so-called *arguments* is denoted and defined by

$$Args_{\mathcal{A}}(\psi) = \{\omega \in \Omega_W : \Phi_\omega \models \psi\} = \Omega_W \setminus [\![\Phi \cup \{\neg\psi\}]\!]^{\downarrow W}, \tag{3}$$

where Φ_ω is obtained from Ω by instantiating all the variables from W according to the partial truth assignment ω [13]. The elements of $Args_{\mathcal{A}}(\neg\psi)$ are sometimes called *counter-arguments* of ψ, see Fig. 2 for an illustration. Note that the elements of $Args_{\mathcal{A}}(\perp)$ are inconsistent with the available evidence Φ, which is why they are sometimes called *conflicts*. The complement of the set of conflicts,

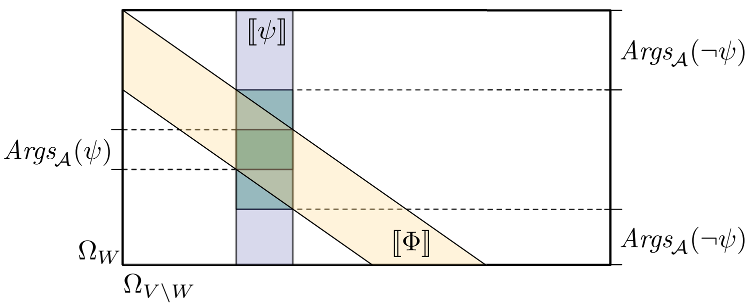

Fig. 2. The sets of arguments and counter-arguments of a hypothesis ψ obtained from the given premises Φ. The sample space Ω_W is a sub-space of the entire space $\Omega_V = \Omega_W \times \Omega_{V\setminus W}$.

$$E_{\mathcal{A}} = \Omega_W \setminus Args_{\mathcal{A}}(\bot) = [\![\Phi]\!]^{\downarrow W}, \tag{4}$$

can thus be interpreted as the available *evidence* in the sample space Ω_W induced by Φ. We will use $E_{\mathcal{A}}$ in its typical role to condition P.

Definition 2. *The* degree of support *of ψ, denoted by $dsp_{\mathcal{A}}(\psi)$, is the conditional probability of the event $Args(\psi)$ given the evidence $E_{\mathcal{A}}$,*

$$dsp_{\mathcal{A}}(\psi) = P(Args_{\mathcal{A}}(\psi)|E_{\mathcal{A}}) = \frac{P(Args_{\mathcal{A}}(\psi)) - P(Args_{\mathcal{A}}(\bot))}{1 - P(Args_{\mathcal{A}}(\bot))}. \tag{5}$$

Definition 3. *The* degree of possibility *of ψ, denoted by $dps_{\mathcal{A}}(\psi)$, is defined by*

$$dps_{\mathcal{A}}(\psi) = 1 - dsp_{\mathcal{A}}(\neg\psi). \tag{6}$$

Note that these formal definitions imply $dsp_{\mathcal{A}}(\psi) \leq dps_{\mathcal{A}}(\psi)$ for all hypotheses $\psi \in \mathcal{L}_V$ and $dsp_{\mathcal{A}}(\psi) = dps_{\mathcal{A}}(\psi)$ for $W = V$. An important property of degree of support is its consistency with pure logical and pure probabilistic inference. By looking at the extreme cases of $W = \emptyset$ and $W = V$, it turns out that degrees of support naturally degenerate into logical entailment $\Phi \models \psi$ and into ordinary posterior probabilities $P(\psi|\Phi)$, respectively. This underlines the theory's pretense of being a unified formal theory of logical and probabilistic reasoning [11].

When it comes to quantitatively evaluate the truth of a hypothesis ψ, it is possible to interpret degrees of support and possibility as respective lower and upper bounds of an interval. The fact that such bounds are obtained without effectively dealing with probability sets or probability intervals distinguishes the theory from most other approaches to probabilistic logic.

3.2 Possible Semantics for the Progic Framework

Now let's turn our attention to the question of interpreting an instance of the progic framework in form of Equation (1) as a probabilistic argumentation system. For this, we will first generalize in various ways the idea of the standard semantics as exposed in Subsection 2.2 to degrees of support and possibility (Semantics 1 to 4). Then we will explore the perspective obtained by considering each attached probability set as an indicator of the premise's reliability (Semantics 5–7). In all cases we will end up with lower and upper bounds for the target interval Y in Equation (1).

Semantics 1: The Generalized Standard Semantics

As in the standard semantics, let each attached probability set X_i be interpreted as a constraint for the possible probability measures, except that we will now restrict the sample space to be a sub-space Ω_W of Ω_V for some fixed set $W \subseteq V$ of probabilistic variables. We use again $\mathbb{P}$ to denote the set of all possible probability measures. Since each premise φ_i defines an event $[\![\varphi_i]\!]^{\downarrow W}$ in Ω_W, we can interpret the set X_i as a constraint $P([\![\varphi_i]\!]^{\downarrow W}) \in X_i$. As before, we use $\mathbb{P}_i = \{P \in \mathbb{P} : P(\varphi_i^{\downarrow W}) \in X_i\}$ to denote[4] the set of all probability measures satisfying the constraint for the i-th premise, and $\mathbb{P}_* = \mathbb{P}_1 \cap \cdots \cap \mathbb{P}_n$ for the combination of all constraints. This leads then to a whole family $\mathbb{A} = \{(V, \mathcal{L}_V, \Phi, W, P) : P \in \mathbb{P}_*\}$ of probabilistic argumentation systems, each of which with its own degree of support (and degree of possibility) function.

To use this interpretation to produce an answer to our main question regarding the extent of the set Y for a conclusion ψ, there are different ways to go. By considering all possible degrees of support, i.e. by defining $Y_1 = \{dsp_{\mathcal{A}}(\psi) : \mathcal{A} \in \mathbb{A}\}$, the first option focuses on degrees of support. As a second option, we may consider the counterpart of the first one with degrees of possibility in its center, from which we get $Y_2 = \{dps_{\mathcal{A}}(\psi) : \mathcal{A} \in \mathbb{A}\}$. As a third alternative, we may consider the minimal degree of support, $\underline{dsp}(\psi) = \min\{dsp_{\mathcal{A}}(\psi) : \mathcal{A} \in \mathbb{A}\}$, and the maximal degree of possibility, $\overline{dps}(\psi) = \max\{dps_{\mathcal{A}}(\psi) : \mathcal{A} \in \mathbb{A}\}$, and use them as respective lower and upper bounds for the target interval $Y_3 = [\underline{dsp}(\psi), \overline{dps}(\psi)]$. Note that in the special case of $W = V$, all three options coincide with the standard semantics as described in Subsection 2.2.

Semantics 2: The Standard Semantics Applied to Degrees of Support

A similar semantics arises, if we consider each set X_i to be a constraint for the degree of support of φ_i. Again, we need to fix a set $W \subseteq V$ of probabilistic variables to get started. Consider then the set $\mathbb{S} = \{dsp_{\mathcal{A}} : \mathcal{A} = (V, \mathcal{L}_V, \Phi, W, P),\ P \in \mathbb{P}\}$ of all possible degree of support functions, the corresponding constraints $\mathbb{S}_i = \{dsp_{\mathcal{A}} \in \mathbb{S} : dsp_{\mathcal{A}}(\varphi_i) \in X_i\}$ for each premise, and the combined constraint $\mathbb{S}_* = \mathbb{S}_1 \cap \cdots \cap \mathbb{S}_n$. As before, we obtain a whole family $\mathbb{A} = \{\mathcal{A} : dsp_{\mathcal{A}} \in \mathbb{S}_*\}$ of probabilistic argumentation systems.

For the determination of the target set Y, we may now consider the same three options as in the first semantics. The story is exactly the same, except that it starts from a different set $\mathbb{A}$. As before, $W = V$ leads in all three cases back to the standard semantics.

[4] We prefer to use the simplified notation $P(\varphi_i^{\downarrow W})$ as an abbreviation for $P([\![\varphi_i]\!]^{\downarrow W})$.

Semantics 3: The Standard Semantics Applied to Degrees of Possibility

By considering each sets X_i as a constraint for the degree of support of φ_i, we obtain another possible semantics for the progic framework. Due to its perfect symmetry to the previous semantics, we will not not discuss it explicitly. Note that we may "simulate" this option by applying the second semantics to the negated premises $\neg\varphi_1^{Y_1}, \ldots, \neg\varphi_n^{Y_n}$, where $Y_i = \{1 - x : x \in X_i\}$ denotes the corresponding set of "negated" probabilities, and vice versa. This string relationships is a simple consequence of the relationship between degrees of support and possibility.

Semantics 4: The Standard Semantics Applied Symmetrically

To obtain a more symmetrical semantics, in which degrees of support and degrees of possibility are equally important, we consider the restricted case where each set $X_i = [\ell_i, u_i]$ is an interval. We may then interpret the lower bound ℓ_i as a sharp constraint for the degree of support and the upper bound u_i as a sharp constraint for the degree of possibility of φ_i. For this, we need again a fixed set $W \subseteq V$ of probabilistic variables to get started. Note that we can use the relationship $dps_{\mathcal{A}}(\psi) = 1 - dsp_{\mathcal{A}}(\neg\psi)$ to turn the two constraints $dsp_{\mathcal{A}}(\psi_i) = \ell_i$ and $dps_{\mathcal{A}}(\psi_i) = u_i$ into two constraints for respective degrees of support or into two constraints for respective degrees of possibility. To obtain a target interval Y for a conclusion ψ, we may then proceed in the same way as in Semantics 2 and 3, the results however will be quite different for all possible options for Y.

Semantics 5: Unreliable Premises (Incompetent Sources)

A very simple, but quite different semantics exists when each premise has a sharp probability $X_i = \{x_i\}$ attached to it. We can then think of x_i to represent the *evidential uncertainty* of the premise φ_i in the sense that φ_i belongs to Φ with probability x_i. Formally, we could express this idea by $P(\varphi_i \in \Phi) = x_i$ and thus interpret Φ as a fuzzy set whose membership function is determined by the attached probabilities.

To make this setting compatible with a probabilistic argumentation system, let us first redirect each attached probability x_i to an auxiliary propositional variable rel_i. The intuitive idea of this is to consider each premise φ_i as a piece of evidence from a possibly unreliable source S_i. The *reliability* of S_i is thus modeled by the proposition rel_i, and with $P(rel_i) = x_i$ we measure its degree of reliability. The subsequent discussion will be restricted to the case of *independent*[5] sources, which allows us to multiply the marginal probabilities $P(rel_i)$ to obtain a fully specified probability measure P over all auxiliary variables.

[5] This assumption may appear to be overly idealized, but there are many practical situations in which this is approximately correct [12, 14]. Relaxing the independence assumption would certainly allow us to cover a broader class of problems, but it would also make the analysis more complicated.

On the purely logical side, we should expect that any statement from a reliable source is indeed true. This allows us to write $rel_i \rightarrow \varphi_i$ to connect the auxiliary variable rel_i with φ_i. With

$$\Phi^+ = \{rel_1 \rightarrow \varphi_1, \ldots, rel_n \rightarrow \varphi_n\}$$

we denote the set of all such material implications, from which we obtain a probabilistic argumentation system $\mathcal{A}^+ = (V \cup W, \mathcal{L}_{V \cup W}, \Phi^+, W, P)$ with $W = \{rel_1, \ldots, rel_n\}$ and P as defined above. This allows us then to compute the degrees of support and possibility for the conclusion ψ and to use them as lower and upper bounds for the target interval Y.

In the proposed setting, only the positive case of a reliable source is modeled, but nothing is said about the behaviour of an unreliable source. For this, it is possible to distinguish between *incompetent* or *dishonest* (but competent) sources. In the case of an incompetent source, from which no meaningful evidence should be expected, we may model the negative behaviour by auxiliary implications of the form $\neg rel_i \rightarrow \top$. Note that these implications are all irrelevant tautologies, i.e. we get back to the same set Φ^+ from above. In this semantics, the values $P(rel_i) = x_i$ should therefore be interpreted as *degrees of competence* rather than degrees of reliability.

Semantics 6: Unreliable Premises (Dishonest Sources)

As before, we suppose that all attached probabilities are sharp values x_i, but now we consider the possibility of the sources being *malicious*, i.e. competent but not necessarily honest. In this case, the interpretation of $P(rel_i) = x_i$ becomes the one of a *degree of honesty* of source S_i. Dishonest sources are different from incompetent sources in their attitude of deliberately stating the opposite of the truth. From a logical point of view, $\neg rel_i$ allows us thus to infer $\neg\varphi_i$, which we may express by additional material implications $\neg rel_i \rightarrow \neg\varphi_i$. This leads to an extended set of premises,

$$\Phi^\pm = \Phi^+ \cup \{\neg rel_1 \rightarrow \neg\varphi_1, \ldots, \neg rel_n \rightarrow \neg\varphi_n\} \equiv \{rel_1 \leftrightarrow \varphi_1, \ldots, rel_n \leftrightarrow \varphi_n\},$$

and a different probabilistic argumentation system $\mathcal{A}^\pm = (V \cup W, \mathcal{L}_{V \cup W}, \Phi^\pm, W, P)$. Note that the difference between the two interpretations may have a huge impact on the resulting degrees of support and possibility of ψ, and therefore produce quite different target sets Y.

Semantics 7: Unreliable Premises (Incompetent and Dishonest Sources)

For the more general case, where each $X_i = [\ell_i, u_i]$ is an interval, we will now consider a refined model of the above-mentioned idea of splitting up reliability into competence and honesty. Let X_i still refer to the reliability of the source, but consider now two auxiliary variables $comp_i$ (for competence) and hon_i (for honesty). This allows us to distinguish three exclusive and exhaustive cases, namely $comp_i \wedge hon_i$ (the source is reliable), $comp_i \wedge \neg hon_i$ (the source is malicious), and $\neg comp_i$ (the source is incompetent). As before, we assume that φ_i holds if S_i is reliable, but also that $\neg\varphi_i$ holds if S_i is malicious. Statements from incompetent sources will again be neglected. Logically, the general behaviour of such a source can thus be modeled by two sentences $comp_i \wedge hon_i \rightarrow \varphi$ and $comp_i \wedge \neg hon_i \rightarrow \neg\varphi_i$, which can be merged into $comp_i \rightarrow (hon_i \leftrightarrow \varphi_i)$. This leads to the set of premises

$$\Phi^* = \{comp_1 \rightarrow (hon_1 \leftrightarrow \varphi_1), \ldots, comp_n \rightarrow (hon_n \leftrightarrow \varphi_n)\}.$$

To turn this model into a probabilistic argumentation system, we need to link the auxiliary variables $W = \{comp_1, \ldots, comp_n, hon_1, \ldots, hon_n\}$ to corresponding probabilities. For this, we assume independence between $comp_i$ and hon_i, which is often quite reasonable. If we assume the least restrictive interval $[0, 1]$ to represent a totally incompetent source, and similarly the most restrictive interval $[x_i, x_i]$ to represent a totally competent source, then $u_i - \ell_i$ surely represents the source's degree of incompetence, from which we obtain

$$P(comp_i) = 1 - (u_i - \ell_i) = 1 - u_i + \ell_i$$

for the marginal probability of $comp_i$. Following a similar line of reasoning, we first obtain $P(comp_i \wedge hon_i) = \ell_i$ for the combined event $comp_i \wedge hon_i$ of a reliable source, which then leads to

$$P(hon_i) = \frac{\ell_i}{P(comp_i)} = \frac{\ell_i}{1 - u_i + l_i}$$

for the marginal probability of hon_i. As before, we can use the independence assumption to multiply these values to obtain a fully specified probability measure P over all auxiliary variables. With $\mathcal{A}^* = (V \cup W, \mathcal{L}_{V\cup W}, \Phi^*, W, P)$ we denote the resulting probabilistic argumentation system, from which we obtain degrees of support and possibility for ψ, the bounds for the target interval Y. Note that $\mathcal{A}^+$ and $\mathcal{A}^\pm$ from the previous two semantics are special cases of $\mathcal{A}^*$, namely for $u_i = 1$ (hon_i becomes irrelevant, and rel_i undertakes the role of $comp_i$) and $\ell_i = u_i$ ($comp_i$ becomes irrelevant, and rel_i undertakes the role of hon_i), respectively.

4 Conclusion

Attaching probabilities to logical sentences is one of the most intuitive and popular starting points for the construction of a probabilistic logic. With the proposed progic framework, for which no particular semantics is imposed, the paper presents a unifying umbrella which covers many existing probabilistic logics. This is the first contribution of the paper.

The second contribution is the discussion of several possible semantics obtained by looking at it as different instances of a probabilistic argumentation system. This underlines the richness and diversity of the common framework.

The discussion also contributes to a better understanding of the connection between the theory of probabilistic argumentation and other probabilistic logics.

This paper is an important partial result in the context of a more comprehensive project, in which other possible semantics and a common computational machinery are currently under investigation.

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