Editorial

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Special issue on Combining Probability and Logic
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How is probability related to logic? Should probability and logic be combined? If so, how?

Bayesianism tells us we ought to reason probabilistically. In that sense, probability theory is logic. How then does probability theory relate to classical logic and the various non-classical logics that also stake a claim on normative reasoning? Is probability theory to be preferred over other logics or vice versa? Is probability theory to be used in some situations, and the other logics in other situations? Or should probability be combined with other logics?

These questions were important in the time of Augustus de Morgan. Indeed de Morgan himself argued that Aristotelian logic was unnecessarily restrictive in scope, and with his contemporary George Boole he began to broaden its horizons, initiating a renaissance in logic. The title of his most important book bears witness to his vision of a comprehensive logic encompassing probability: "Formal Logic; or the calculus of Inference, Necessary, and Probable".

While the above questions are not new, we now urgently require some answers. Artificial intelligence is one key discipline in which probability theory competes with other logics for application. It is becoming vitally important to evaluate and integrate systems that are based on very different approaches to reasoning, and there is strong demand for theoretical understanding of the relationships between these approaches.

The aim of this volume is to address the relationship between probability and logic from an interdisciplinary perspective. I hope that the themes

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presented here will be of interest to mathematicians, logicians, philosophers, computer scientists and engineers.

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The papers presented here elucidate two key ways in which one can tackle the question of the relationship between probability and logic. One approach is to argue that *probability is logic*, which requires showing that probability is a determinate relation between statements. Kyburg, Howson and Paris and Vencovská appeal to the concepts of frequency, consistency and entropy respectively to determine this relation. Alternatively one can explore other formalisms which *interface between probability and logic*: argumentation in the case of Fox and Kohlas; default reasoning in the case of Bourne and Weydert.

In 'Are There Degrees of Belief?', Henry Kyburg assesses John Maynard Keynes' view that probability is a logical relationship between premiss and conclusion: the degree to which the premiss entails the conclusion. How do we ascertain such probabilities? Keynes suggested that we intuit them, and that we can also apply the principle of indifference to measure them. The appeal to intuition was attacked by Frank Ramsey, while the principle of indifference faces a number of difficulties and Keynes accepted that its use is at best limited. Kyburg argues that we can use frequencies to determine logical probabilities and endorses Keynes' view that probabilities need not be point-valued: the frequency approach leads naturally to interval-valued probabilities.

Ramsey developed his own logical view of probability, arguing that degrees of belief must satisfy the laws of probability on pain of inconsistency. Colin Howson develops this position in 'Probability and Logic', emphasising the parallels between probability and deductive logic, and arguing in favour of a unified conception of logic based around the notion of consistency: 'the logic of consistent assignments of *truth-values* subject to the usual classical truth-definition constraints is deductive logic; the logic of consistent assignments of *uncertainty-values*, subject to the appropriate constraints on these, will be probabilistic logic.'

Rudolf Carnap developed Keynes' idea that premisses determine the probability of a conclusion but struggled to identify the 'logical' probability function that relates premiss and conclusion, narrowing it down only to a continuum of probability functions. Edwin Jaynes proposed the maximum entropy function as the logical probability function if the domain is finite (this is the probability function that represents the premisses but is otherwise maximally non-committal, degree of non-commitment or uncertainty normally being measured by entropy), but he could only extend this proposal to infinite domains in problems in which there are a number of obvious invariances. In 'The Emergence of Reasons Conjecture', Jeff Paris and Alena Vencovská conjecture (i) by taking the limit of maximum entropy functions on finite logical languages, one can identify a logical probability function on a logical language containing finitely many propositional variables, finitely many monadic predicates but infinitely many constants; (ii) whatever the actual form of the premisses, the logical function is the same as one derived from a 'complete set of reasons', i.e. constraints that take the form of probabilities of instantiated predicates conditional on mutually exclusive and exhaustive hypotheses (the reasons). Paris and Vencovská prove this conjecture for the case in which there is a single monadic predicate and premisses involving no more than two constants.

While for Howson consistency provides an umbrella under which deductive logic and probabilistic logic shelter, for John Fox argumentation plays this role. Arguments are the bread and butter of logic, and they are deductive or inductive according to whether they have certain or uncertain premisses and conclusions. Fox argues in 'Probability, Logic and the Cognitive Foundations of Rational Belief' that quantitative probabilistic arguments do not exhaust the realm of the inductive — uncertain reasoning may be qualitative or semi-quantitative and it may be non-probabilistic, dealing with other '*p*-modals' such as possibility and plausibility. Fox provides examples of the argumentation approach applied to medical decision making, and uses non-classical logic to formalise a logic of argument.

Jürg Kohlas takes the argumentation approach as his starting point in 'Probabilistic Argumentation Systems: a New Way to Combine Logic with Probability'. While most frameworks for argumentation weigh up qualitative arguments in favour of or against a hypothesis, Kohlas' approach is quantitative: the reliability of an argument is measured probabilistically and then the arguments are aggregated to measure the degree of support of a hypothesis. The arguments themselves are presented in the language of 'information systems', which generalises several formalisms including propositional logic and systems of linear equations. 'Information algebras' are then used to represent and aggregate the probabilities which attach to arguments. The way probability is handled in Kohlas' formalism bears a natural correspondence with the Dempster-Shafer approach to uncertainty.

Default reasoning provides another bridge between probability and logic. Reasoning to a conclusion which holds only by default is a qualitative or semi-quantitative logical process (probabilities are not used explicitly), although it is non-deductive. In fact default reasoning behaves as an inductive logic under the maximum entropy semantics presented by Rachel Bourne in 'Explaining Default Intuitions using Maximum Entropy'. Here the default conclusion is the conclusion that is most probable according to the maximum entropy probability function. In fact Bourne advocates an extension of the maximum entropy approach which allows premisses and conclusions to have variable strengths associated with them. This system, Bourne argues, can be used as a benchmark with which to evaluate intuitions behind other default logics.

Bourne's variable strengths allow defaults to be ranked in order of strength. In 'System JLZ — Ranking Default Reasoning by Minimal Ranking Constructions', Emil Weydert uses rankings to provide an alternative framework for default reasoning. There is a natural correspondence between rankings and (non-standard) probability assignments and so probabilistic operations such as conditionalisation induce corresponding ranking operations. Weydert extends Wolfgang Spohn's ranking conditionalisation to his own 'J' framework but argues that the resulting default conclusions are too cautious and so introduces his 'JLZ' system. Weydert compares his approach to other strategies for default reasoning and argues that the maximum entropy approach is too sensitive to small changes in strengths of defaults.

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The papers in this volume were submitted by participants at the 4th Augustus de Morgan Workshop 'Combining probability and logic', held at King's College London on 4th-6th November 2002, and organised by philosophy.ai (the philosophy and artificial intelligence research group in the Philosophy Department) and the Group of Logic and Computation in the Computing Department, King's College London. Thanks to the speakers and all those who attended this workshop, as well as to Jane Spurr and Anna Maros for invaluable help in organising it.