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One philosopher's modus ponens is another's modus tollens: pantomemes and nisowir

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Abstract

That one person's modus ponens is another's modus tollens is the bane of philosophy, I argue, because it strips many philosophical arguments of their persuasive force. The problem is that philosophical arguments become mere ***pantomemes***: arguments that are reasonable to resist simply by denying the conclusion. I show that appeals to proof, intuition, evidence and truth fail to alleviate the problem. However, I develop two broad strategies that do help in certain circumstances: an appeal to ***normal informal standards of what is reasonable*** (nisowir) and ***argument by interpretation***. The method of explication features prominently in both strategies, and I extend this method to apply to nisowir, introducing the concept of ***canonical explication***. I illustrate the two strategies with examples of arguments from formal epistemology, and I suggest that an appeal to nisowir might help to defend against philosophical scepticism by shifting the burden of proof to the sceptic.

§1 Introduction

Recall the two rules of inference, modus ponens (MP) and modus tollens (MT):

$$\text{MP} \qquad \begin{array}{r} \theta \to \varphi \\ \theta \\ \hline \varphi \end{array} \qquad\qquad \text{MT} \qquad \begin{array}{r} \theta \to \varphi \\ \neg\varphi \\ \hline \neg\theta \end{array}$$

That one philosopher's modus ponens is another's modus ponens (henceforth, MP/MT) is the phenomenon that when one philosopher uses modus ponens to argue for some conclusion φ by appeal to $\theta \to \varphi$ and θ, another might reasonably respond by simply denying φ and using modus tollens to undermine θ.[1]

Here is an example of Putnam's, which opens by discussing Quine's argument for ontological relativity:

[1] Here θ and/or φ may be logically complex propositions. Throughout the paper I follow standard practice in using 'modus ponens' and 'modus tollens' to refer either to rules of inference or to particular arguments that can be construed as appealing to applications of those rules of inference. Which usage is intended will be clear from the context.

even *within* my language, or rather, within my metalanguage, I can define truth and reference in such a way that

> "Rabbit" refers to rabbits in my object language

comes out true, or in such a way that

> "Rabbit" refers to mereological complements of rabbits in my object language

comes out true, without altering the set of true sentences of my object language in any way, and without altering the truth conditions for observation sentences as wholes; and in such a case, Quine tells us, there is no fact of the matter as to which reference/truth definition is correct. …

In *Reason, Truth, and History* I used an argument similar to Quine's, but drew an opposite conclusion (thus illustrating the well known maxim that one philosopher's *modus ponens* is another philosopher's *modus tollens*). I argued there that metaphysical realism leaves us with no intelligible way to refute ontological relativity, and concluded that metaphysical realism is wrong. And I still see ontological relativity as a refutation of any philosophical position that leads to it. (Putnam, 1994, p. 280.)

Putnam takes the modus tollens route in response to Quine's modus ponens:

MP (Quine)	MT (Putnam)
$\dfrac{MR \to OR \quad MR}{OR}$	$\dfrac{MR \to OR \quad \neg OR}{\neg MR}$

Here MR stands for metaphysical realism and OR for ontological relativity.

It is characteristic of the MP/MT phenomenon that both parties agree with respect to the first, conditional premiss. The disagreement arises with respect to the second premiss and the conclusion. Note that nothing hangs on which party advocates the modus ponens and which advocates the modus tollens. By taking the contrapositive of the first premiss, Putnam's argument can be viewed as an instance of MP, and Quine's an instance of MT:

MP (Putnam)	MT (Quine)
$\dfrac{\neg OR \to \neg MR \quad \neg OR}{\neg MR}$	$\dfrac{\neg OR \to \neg MR \quad MR}{OR}$

Thus there is nothing in the logical form of these inferences to arbitrate between them. MP/MT does not admit a logical resolution.

MP/MT extrapolates beyond philosophy. For example, one can caricature some of the debate in the run-up to the 2020 US presidential election as follows, where TB stands for 'Trump is to be believed' and AG for 'America is great again':

MP (pro Trump)	MT (anti Trump)
$\dfrac{TB \to AG \quad TB}{AG}$	$\dfrac{TB \to AG \quad \neg AG}{\neg TB}$

However, MP/MT is not universally generalisable: it is not always reasonable to respond to an argument by modus ponens simply by denying the conclusion. Many arguments in mathematics, for example, can be cast as instances of modus ponens and are not so easy to resist when $\theta \to \varphi$ and θ are supported by valid mathematical proofs. For example, suppose *PI* says of a particular structure *I* that *I* is a prime ideal in Boolean algebra $\mathscr{A}$, and *MI* says that *I* is a maximal ideal in Boolean algebra $\mathscr{A}$. The following modus ponens would not be easy to resist, where a proof is provided for both premisses:

MP (ideals)	MT (not viable)
$\dfrac{PI \to MI \quad PI}{MI}$	$\dfrac{PI \to MI \quad \neg MI}{\neg PI}$

The concern arises, then, that MP/MT poses a particular problem for philosophy. The aim of this paper is to clarify why MP/MT poses a problem for philosophy and to develop some strategies for resolving the problem in certain circumstances. In §2 I argue that MP/MT poses a problem for the public practice of philosophy, because it makes it difficult for someone with no prior opinion about a conclusion of a philosophical argument to reach a reasoned opinion. I also distinguish this problem from standard epistemological scepticism. In §3 I observe that arguments for the second premiss of the MP or MT need to bottom out in some suitable form of public justification. Hence, I argue, appeals to philosophical intuition, evidence, truth, proof and further philosophical argument fail to solve the MP/MT problem. In §4, I discuss two examples from the area of formal epistemology: Cox's argument for probabilism and the argument of Hawthorne et al. (2017) for the Principle of Indifference. These arguments illustrate the limitations of mathematical proof in addressing MP/MT. I then develop two broad strategies that can help in some cases: an appeal to normal informal standards of what is reasonable (§5), and argument by interpretation (§6). I conclude in §7 that these strategies require a shift in philosophical methodology towards a more prominent role for empirical justification and explication.

§2
Why MP/MT poses a problem

MP/MT poses a problem because it raises the worry that very many philosophical arguments are what I will call 'pantomemes': arguments (often enthymemes) that are reasonable to resist simply by denying the conclusion. Schematically:

Oh yes it is!	Oh no it isn't!
$\dfrac{\ldots \quad \ldots}{\varphi}$	$\dfrac{\ldots \quad \neg\varphi}{\ldots}$

The concern is that very many philosophical arguments are instances of modus ponens, and, thanks to MP/MT, they are thereby pantomemes.

If very many philosophical arguments are pantomemes then philosophy is in trouble. Suppose a third party who has no prior opinion about θ or φ wants to find out whether φ. Philosophers would point her to relevant arguments for and against φ. However, if for each φ-argument there is a reasonable $\neg\varphi$-argument, and vice versa, then philosophical arguments fail to provide grounds for preferring one of φ and $\neg\varphi$ over the other. It thus seems practically impossible for our interested third party to come to a reasoned opinion about φ. Philosophy would at best be an amusing diversion—like a pantomime—not a reliable means of reaching reasoned opinions.

This worry applies to very many philosophical arguments, for the following reasons. Firstly, as I shall explain below, philosophical arguments can be cast as (possibly enthymematic) inferences by modus ponens. Second, philosophical arguments are of a kind that can easily be challenged by MP/MT. Consider this second point. A philosophical argument is of interest to the extent that its conclusion is substantive and controversial. A controversial conclusion can be denied, however, and this opens the door to MP/MT.[2] The only way to resist MP/MT would be to provide a justification of the premisses of the modus ponens, of a sort that does not also apply to the premisses of the modus tollens. In the case of mathematics, justification takes the form of proof from generally accepted axioms and definitions. But this avenue is not usually available to philosophical arguments. Even in the area of formal philosophy, philosophical premisses are usually not all purely mathematical, and so not amenable to rigorous proof, as I will illustrate in §4. Moreover, it is hard to find generally accepted starting points in philosophy from which to begin proofs.

That philosophical arguments can be cast as arguments by modus ponens can be seen as follows. If an argument is intended to be deductive—i.e., if the negation of the conclusion φ is taken to be incompatible with the conjunction θ of the premisses—and the argument is not already in the form of an argument by modus ponens, then the conditional $\theta \to \varphi$ can be taken to be an additional implicit premiss, subsuming it into the form of MP as set out above. If, on the other hand, the philosophical argument is intended to be inductive, i.e., to render the conclusion plausible, then it can be cast as an instance of some inductive version of modus ponens and is susceptible to MP/MT, as I shall now show. For simplicity of exposition, we shall focus here on the case in which θ and φ are distinct atomic propositions and there is no further background information.

Any probabilistic logic validates the following inductive versions of MP and MT:

MP	MT
$\dfrac{\begin{array}{r}\theta \to \varphi\\ \theta^{0.9}\end{array}}{\varphi^{[0.9,1]}}$	$\dfrac{\begin{array}{r}\theta \to \varphi\\ \neg\varphi^{0.9}\end{array}}{\neg\theta^{[0.9,1]}}$

Here the modus ponens is to be read: if $\theta \to \varphi$, and θ has probability 0.9, then φ has probability at least 0.9. Similarly for the modus tollens.[3]

[2] Indeed, it is sometimes claimed in jest that for each consistent philosophical position, and for many an inconsistent position, there exists some philosopher who advocates it. Insofar as there is more than a grain of truth to that maxim, one philosopher's modus ponens really is another's modus tollens.

[3] Here the first premiss—the conditional—is taken to be certain, although this assumption can be

For example, objective Bayesian inductive logic performs inferences using the probability function, from all those that satisfy constraints imposed by the premisses, that has maximum entropy (Williamson, 2017). Objective Bayesian inductive logic validates the following versions of MP and MT:[4]

MP	MT
$\dfrac{\theta \to \varphi \quad \theta^{0.9}}{\varphi^{0.95}}$	$\dfrac{\theta \to \varphi \quad \neg\varphi^{0.9}}{\neg\theta^{0.95}}$

The above versions of MP and MT cover the case in which the conclusion is less than certain because the second premiss is less than certain. Alternatively, the conditional first premiss may be less than certain. In any probabilistic logic,

MP	MT
$\dfrac{\theta \to \varphi^{0.9} \quad \theta}{\varphi^{0.9}}$	$\dfrac{\theta \to \varphi^{0.9} \quad \neg\varphi}{\neg\theta^{0.9}}$

In each case, the first premiss attaches probability 0.9 to the conditional $\theta \to \varphi$.[5] Note that in the context of inductive logic, it is often more natural to consider conditional probability, rather than the probability of a material conditional. But the MP/MT problem also arises when conditional probabilities are used:

MP	MT
$\dfrac{\varphi\vert\theta^{0.9} \quad \theta}{\varphi^{0.9}}$	$\dfrac{\varphi\vert\theta^{0.9} \quad \neg\varphi}{\neg\theta}$

Here the modus ponens is to be read: if the probability of φ conditional on θ is 0.9 and θ is true then φ has probability at least 0.9.[6] Note that the modus tollens yields a stronger conclusion than the modus ponens in this case. The two kinds of uncertainty can be combined, as follows.

relaxed, as we shall see below. There is nothing special about the value 0.9 here. To see this, consider for example the modus ponens. If the second premiss were θ^x, which attaches probability $x \in [0,1]$ to θ, then one could calculate the probability of the conclusion as follows. The first premiss forces probability 0 on the state $\theta \wedge \neg\varphi$. Probability x must thus be given to the remaining θ-state, $\theta \wedge \varphi$. The remaining probability, $1-x$, must then be divided between the two remaining states, $\neg\theta \wedge \varphi$ and $\neg\theta \wedge \neg\varphi$. Thus the probability of φ, which is the sum of the probabilities of $\theta \wedge \varphi$ and $\neg\theta \wedge \varphi$, must be between x and 1.

[4] In the case of the modus ponens, for instance, this is because the maximum entropy probability function gives the states $\neg\theta \wedge \varphi$ and $\neg\theta \wedge \neg\varphi$ the same probability, 0.05.

[5] Consider the MP, for example. The first premiss forces probability 0.1 on $\theta \wedge \neg\varphi$ and the second premiss then ensures that $\theta \wedge \varphi$ receives all the remaining probability, 0.9. φ is consistent only with the latter state, and so must receive probability 0.9.

[6] The conditional probability imposes a constraint on unconditional probabilities, namely, $P(\varphi \wedge \theta) = 0.9P(\theta)$. We do not make the further assumption here that the conditional probability is *defined* by unconditional probabilities via the formula $P(\varphi|\theta) \stackrel{\text{df}}{=} P(\varphi \wedge \theta)/P(\theta)$. Such a definition would not be compatible with the modus tollens, which gives θ zero probability. To see this, note that for the MT, $P(\varphi) = 0$ so $0 = P(\varphi \wedge \theta) = 0.9P(\theta)$ and hence $P(\theta) = 0$.

MP	MT
$\dfrac{\varphi\vert\theta^{0.9} \quad \theta^{0.9}}{\varphi^{[0.81,1]}}$	$\dfrac{\varphi\vert\theta^{0.9} \quad \neg\varphi^{0.9}}{\neg\theta^{[0.89,1]}}$

Again, the modus tollens yields a stronger conclusion than the modus ponens.

In sum, philosophical arguments can straightforwardly be cast as arguments by modus ponens, and MP/MT applies to both deductive and inductive arguments by modus ponens.

It is worth observing that MP/MT is different to the standard problem of epistemological scepticism: it does not challenge the possibility of knowledge of the external world. The MP and MT parties may both think they know the conclusions of their arguments by virtue of their argumentation, and one of them may indeed be right. The problem is for an interested third party who has no prior opinion about θ or φ—how is she to be convinced one way or the other?

MP/MT is arguably a more serious problem than standard scepticism, for three key reasons. Firstly, MP/MT is of practical as well as theoretical concern. There are few, if any, thoroughgoing philosophical sceptics who disavow all knowledge of the external world, yet there are many instances of arguments by modus tollens being used as serious responses to arguments by modus ponens, and vice versa. Second, MP/MT applies not only to claims about the external world, but also to conclusions that are beyond the scope of standard scepticism: conclusions about the internal world, metaphysics, ethics, etc. Third, scepticism is itself susceptible to MP/MT. Let K stand for 'I know that', B for 'I am a brain in a vat' and H for 'this is a human hand in front of me'. Consider:

MP (scepticism)	MT ('common sense')
$\neg K \neg B \to \neg KH$	$\neg K \neg B \to \neg KH$
$\neg K \neg B$	KH
$\neg KH$	$K \neg B$

Here the modus ponens represents the sceptical inference, while the modus tollens captures the 'common-sense' response to scepticism. (This response is often attributed to Moore (1925), although Baldwin (1990, §9.4) argues against this interpretation of Moore.)

Scepticism has been described as a scandal for philosophy (Kant, 1781, B xxxix). Now a scandal can be a good thing because it can spark interest and engagement. MP/MT, on the other hand, is perhaps the bane of philosophy: it seems to undermine the whole public enterprise of philosophy.

§3
Intuition and public justification

The common-sense response to scepticism is normally regarded as rather weak. However, the claim that when one apparently holds out one's hand one knows it is a human hand does have the merit of agreeing with intuition. The question thus arises as to whether an appeal to intuition can mitigate the MP/MT problem.

There are, after all, many advocates of appeals to intuitions in philosophy, including Bealer (1998, 2000); Williamson (2004) and Cath (2012), for example. If the second premiss of the modus ponens were more intuitive than that of the modus tollens, or vice versa, intuition would appear to favour one of the arguments over the other.

The difficulty here is that philosophical intuitions are notoriously subjective and simply denying such an intuition is usually a reasonable response. Consider, for example, Ramsey's response to Keynes' intuition that there are logical probability relations between propositions:

> But let us now return to a more fundamental criticism of Mr. Keynes' views, which is the obvious one that there really do not seem to be any such things as the probability relations he describes. He supposes that, at any rate in certain cases, they can be perceived; but speaking for myself I feel confident that this is not true. I do not perceive them, and if I am to be persuaded that they exist it must be by argument; moreover I shrewdly suspect that others do not perceive them either, because they are able to come to so very little agreement as to which of them relates any two given propositions. … If … we take the simplest possible pairs of propositions such as 'This is red' and 'That is blue' or 'This is red' and 'That is red', whose logical relations should surely be easiest to see, no one, I think, pretends to be sure what is the probability relation which connects them. (Ramsey, 1926, pp. 161–2.)

More generally, it is usually reasonable to respond to a philosophical argument by modus ponens that is grounded in intuition by putting forward a modus tollens which appeals to conflicting intuitions:

MP	MT
$\theta \to \varphi$	$\theta \to \varphi$
Intuitively: θ	Intuitively: $\neg\varphi$
φ	$\neg\theta$

In fact, the proponent of the modus tollens does not even need to take $\neg\varphi$ to be intuitive—she may have other reasons for doubting φ.[7] Since the first premiss, $\theta \to \varphi$, is not under contention, this doubt about the conclusion φ of the modus ponens must extend to the second premiss, θ, however benign it may seem. (As we saw above, this is true of inductive as well as deductive arguments.) Hence, the MT proponent can reason as follows: I have grounds for denying φ, so, even though θ does seem benign, it must be a wolf in sheep's clothing, i.e., a strong premiss formulated in such a way as to seem innocuous; therefore it should not trouble me that θ seems intuitive. We see, then, that the modus tollens can be a reasonable response to the modus ponens even where θ seems intuitive to the MT proponent and $\neg\varphi$ does not. It is enough that the MT proponent has some other grounds for denying φ.

This consideration motivates a more general concern about the appeal to intuition. The worry is that it makes much philosophical argumentation look like

[7] Presumably φ itself is not intuitively true—otherwise there would be no need for the MP proponent to argue for φ from a premiss that is supported by intuition. Thus, these other reasons are unlikely to conflict with intuitions about φ.

artifice: i.e., as trying to devise arguments by modus ponens in such a way that θ appears benign, even though from a logical point of view it is as strong as φ and should be as contentious as φ.[8]

We need to look further to find some means to resolve the MP/MT problem. It should by now be clear that to favour MP over MT we would need a form of public justification that applies to θ but not to $\neg\varphi$. On the other hand, to favour MT over MP this kind of justification would need to apply to $\neg\varphi$ and not to θ. Thus what we seek is an appropriate form of public justification. This would need to be public in the sense that it is available to all participants in the debate and objective enough to determinately favour one side over the other.

We saw that mathematical proof from a generally accepted starting point plays the required role in mathematics, but this form of public justification is seldom directly applicable to philosophical arguments, because it is rarely the case that all the premisses of a philosophical argument are purely mathematical, and because it is hard to find generally accepted starting points in philosophy. One might suggest that philosophical argument provides the required form of public justification, by analogy with mathematical proof. However, the problem of starting points remains. Unless there is a generally accepted starting point, we have regress: a philosophical argument for the second premiss θ will itself be susceptible to MP/MT.

I noted that an appeal to intuition is unsuccessful because intuition is personal and subjective, and it is reasonable to doubt a premiss that yields a contentious conclusion, even if the premiss is intuitive. Instead, one might suggest an appeal to evidence rather than intuition: the idea is that if θ is evidence and $\theta \to \varphi$ is uncontentious, then φ can be established. Like intuition, however, evidence is personal, and though θ may be evidence for the MP proponent, it will not be evidence for the MT proponent, who argues that $\neg\theta$. This is the case under all the usual accounts of evidence—e.g., evidence as knowledge (Williamson, 2000), what is truly believed (Mitova, 2017), what is rationally granted (Williamson, 2015), or what is possessed as information (Rowbottom, 2014). For example, if evidence is knowledge and θ is evidence for the MP proponent then θ is believed by the MP proponent; however, it is not believed by the MT proponent, who argues against θ, so it is not evidence for the MT proponent. Hence that the MP proponent takes θ as evidence will not trouble the MT proponent. Likewise, even if $\neg\varphi$ is evidence for the MT proponent, it will not be evidence for the MP proponent, so the modus ponens is a reasonable response to the modus tollens.

Thus far I have suggested that intuition, mathematical proof, philosophical argument and evidence each fail to provide the sort of public justification that could resolve the MP/MT problem. It is worth noting that an appeal to truth also fails to solve the MP/MT problem. Admittedly, if the two premisses of the modus ponens are true, then the conclusion of the modus tollens must be false, and vice versa. So there is a sense in which truth does adjudicate between the two arguments.[9] However, that at most one of these two arguments can be sound does not imply that both cannot be reasonable. The problem remains that each argument offers

[8]We see an example of this in the next section, in relation to Cox's theorem. Cox originally presented his key premisses in a benign way, but when his argument was made rigorous, it became clear that the required premisses were no more benign than the conclusion.

[9]Under those views of evidence that take evidence to be factive, evidence adjudicates between the two arguments in a similar way. Under such views, if θ is evidence for the MP proponent and $\theta \to \varphi$ then φ is true, so $\neg\varphi$, being false, is not evidence for either party.

a reasonable response to the other, where the truth of the second premiss of each argument is open to dispute. Moore (1939, p. 149), for example, suggests that it is enough to know H (i.e., that this is a hand in front of me) for his response to scepticism to go through. However, this is precisely what is at issue: the sceptic disavows this knowledge. Hence the truth of KH cannot publicly arbitrate between the sceptical modus ponens and the 'common-sense' modus tollens discussed above.

In response to these concerns, one might take issue with the claim that we need any kind of public justification at all. Perhaps philosophy is a personal journey and not in fact concerned with public persuasion. Perhaps the task of each philosopher is to add to the stock of arguments in the literature and to believe those propositions that best cohere with, best explain, or best unify, these arguments, her evidence and her intuitions. It is compatible with this holistic enterprise that no philosophical argument is persuasive simpliciter, but only relative to the entire literature and a particular philosopher's evidence and intuitions.

If this personalist response is correct, philosophy is necessarily elitist: only those who have mastered everything are in a position to form a view about anything. The personalist might bite the bullet here and accept that reasonable opinions are hard to come by in philosophy. But a further worry remains. The personalist response is also necessarily subjective: for any view, it may be reasonable to take the opposite view, provided one has sufficiently different intuitions and evidence. Thus, the personalist response merely concedes the point—it does not solve the MP/MT problem. Philosophical arguments remain pantomemes under this view. The fact is that philosophical arguments are usually presented as persuasive in their own right, and MP/MT threatens to undermine the conception of philosophy as a public practice that incrementally establishes claims by means of persuasive arguments.

§4
Examples

Let us now consider two examples from the area of formal epistemology. These examples will help to illustrate the limitations of the use of mathematical proof in philosophical arguments. §5 and §6 will go on to use these examples to develop some more viable responses to the MP/MT problem.

The first example is known as Cox's theorem. This argument was originally put forward by Cox (1946). It takes the form of an argument by modus ponens, with first premiss $CS \to CP$ where CP is the claim that conditional credences are isomorphic to conditional probabilities and CS is a conjunction of 'common-sense' conditions. Cox claims that these common-sense conditions hold, and he concludes that conditional credences are isomorphic to conditional probabilities. Cox's argument is one of an array of arguments for probabilism, i.e., the view that rational degrees of belief are probabilities.

Now, Cox's original argument was invalid (Halpern, 1999). In order to provide a rigorous argument, one needs to strengthen the common-sense conditions. Paris (1994, p. 24) provided a rigorous version, which can be stated as follows, where $S\mathscr{L}$ is the set of sentences of a propositional language $\mathscr{L}$:

Theorem 1 (*Cox/Paris*). *Suppose that a conditional belief function* $Bel(\theta|\psi) : S\mathscr{L} \times S\mathscr{L} \longrightarrow [0,1]$ *is defined for each consistent* ψ *and each* θ*, that* $\varphi \wedge \psi$ *is consistent, and*

1. *if* θ *is logically equivalent to* θ' *and* ψ *is logically equivalent to* ψ' *then* $Bel(\theta|\psi) = Bel(\theta'|\psi')$,

2. *if ψ logically implies θ then* $Bel(\theta|\psi) = 1$ *and* $Bel(\neg\theta|\psi) = 0$,

3. *there is some continuous function* $F : [0,1] \times [0,1] \longrightarrow [0,1]$ *which is strictly increasing in both coordinates on* $(0,1] \times (0,1]$ *such that* $Bel(\theta \wedge \varphi|\psi) = F(Bel(\theta|\varphi \wedge \psi), Bel(\varphi|\psi))$,

4. *there is some decreasing function* $S : [0,1] \longrightarrow [0,1]$ *such that* $Bel(\neg\theta|\psi) = S(Bel(\theta|\psi))$,

5. *for any* $a, b, c \in [0,1]$ *and* $\epsilon > 0$ *there are* $\theta_1, \theta_2, \theta_3, \theta_4 \in S\mathscr{L}$ *such that* $\theta_1 \wedge \theta_2 \wedge \theta_3$ *is consistent and* $Bel(\theta_4|\theta_1 \wedge \theta_2 \wedge \theta_3), Bel(\theta_3|\theta_1 \wedge \theta_2), Bel(\theta_2|\theta_1)$ *are within* ϵ *of* a, b, c *respectively.*

Then there is a continuous, strictly increasing, surjective function $g : [0,1] \longrightarrow [0,1]$ *such that* $gBel(\cdot|\tau)$ *is a probability function on* $\mathscr{L}$*, for any tautology* τ.

While a mathematical proof does indeed provide a public justification for this theorem, the theorem itself constitutes only the first premiss, $CS \rightarrow CP$, of the modus ponens. There is ample room for someone who denies the conclusion to deny one or more of the conditions 1–5, or one of the other presuppositions of the theorem. For example, Shafer (2004) takes issue with 3 and 4, and also the condition that there is a real-valued belief function. Colyvan (2004, 2008), on the other hand, denies the presupposition that this all takes place in classical logic. Conditions 1 and 2 would be challenged by anyone who thinks it too much to require that the agent in question be logically omniscient. Condition 5, which requires that the agent's credences assume denumerably many different values, is also contestable. Hence, despite the existence of a mathematical proof, the second premiss of the modus ponens lacks a suitable public justification, opening the door to MP/MT:

MP (Cox)	MT (anti Cox)
$CS \rightarrow CP$	$CS \rightarrow CP$
CS	$\neg CP$
CP	$\neg CS$

Let us turn to a second example from the area of formal epistemology. Hawthorne et al. (2017) argued that the Principal Principle implies the Principle of Indifference, which we can write as $PP \rightarrow PoI$. Here PoI is a version of the Principle of Indifference which says that one should believe a contingent atomic proposition to degree $\frac{1}{2}$, in the absence of any evidence that bears on the truth of that proposition. David Lewis' Principal Principle says that one should calibrate a degree of belief to a chance, given that chance and other admissible information. The Principal Principle needs to be accompanied by auxiliary conditions that specify facts about admissibility. Here is the key result:

Theorem 2 (*Hawthorne et al.*). *Suppose* F *is a contingent atomic proposition,* $0 < x < 1$, *and:*

Principal Principle. $P(A|XE) = x$, *where* X *says that the chance* A *is* x *and* E *is admissible.*

Condition 1. If E *is admissible and* XE *contains no information that renders* F *relevant to* A*, then* EF *is admissible.*

Condition 2. If E is admissible and XE contains no information relevant to F, then $E(A \leftrightarrow F)$ is admissible.

Then $P(F|XE) = 1/2$ whenever E is admissible and XE contains no information pertaining to F or its relevance to A.

This result forms the conditional $PP \rightarrow PoI$, where PoI is the claim that $P(F|XE) = 1/2$ whenever E is admissible and XE contains no information pertaining to F or its relevance to A, and where PP encompasses the Principal Principle, Conditions 1 and 2, and other presuppositions, including probabilism. (Note that this argument is directed at advocates of standard Bayesianism, in which probabilism is taken for granted.)

Again, there are two ways one can argue here. One can maintain with Hawthorne et al. (2017) that PP is well motivated and conclude PoI (the modus ponens), or one can endorse the modus tollens and argue that PoI being false tells against PP. Pettigrew (2020), Titelbaum and Hart (2020) and other detractors might be interpreted as taking this latter route, finding fault with Conditions 1 and 2 in particular.

MP (Hawthorne et al.)	MT (anti Hawthorne et al.)
$PP \rightarrow PoI$	$PP \rightarrow PoI$
PP	$\neg PoI$
PoI	$\neg PP$

In the next section I develop a strategy for telling between these two arguments and we shall see that it provides just the sort of public justification that promises to resolve the MP/MT problem in certain cases.

These two examples typify the use of proof in formal philosophy. It is typical that mathematical proof provides a public justification for the first, conditional premiss of an argument. (In some cases—e.g., Cox's theorem—the initial proof is invalid. However, as in Cox's case, the result can sometimes be reformulated and made valid by strengthening the second premiss.) At this stage the first premiss becomes uncontentious, but the MP/MT problem then arises. This is because the substantive second premiss usually cannot be wholly justified by mathematical proof. Thus mathematical proof tends to be of limited use in resisting the MP/MT problem in philosophy.

§5
Normal informal standards of what is reasonable

In this section and the next I put forward some positive proposals for tackling the MP/MT problem. In §5.1 I introduce normal informal standards of what is reasonable, or 'nisowir' for short. We see in §5.2 that an appeal to nisowir can help to resolve the MP/MT problem. §5.3 notes that this resolution is intended to be burden-shifting and discusses concerns relating to circularity. In §6 we turn to another strategy for addressing MP/MT, namely argument by interpretation.

§5.1. Explicating nisowir

Let us consider the example of the Principal Principle and the Principle of Indifference in more detail. When defending the argument of Hawthorne et al. (2017) against critics, Landes et al. (2021) argue that one can motivate the second premiss *PP* by appealing to normal informal standards of what is reasonable.

As Landes et al. (2021) explain, David Lewis motivated his Principal Principle by means of a questionnaire, which begins as follows:

> First question. A certain coin is scheduled to be tossed at noon today. You are sure that this chosen coin is fair: it has a 50% chance of falling heads and a 50% chance of falling tails. You have no other relevant information. Consider the proposition that the coin tossed at noon today falls heads. To what degree would you now believe that proposition?
>
> Answer. 50%, of course. … (Lewis, 1980, p. 84.)

The questionnaire provides helpful motivation because:

> we have some very firm and definite opinions concerning reasonable credence about chance. These opinions seem to me to afford the best grip we have on the concept of chance. (Lewis, 1980, p. 84.)

Landes et al. (2021, §2.1) provide a similar questionnaire:

> Imagine you are a goat farmer, interested in the colour of the next goat to be born to your herd. Your evidence determines that the chance of that goat—Ashley, say—being brown (A) is 0.7. Consider three alternative scenarios, and ask yourself in each case what degree of belief in A would be reasonable:
>
> *(a)* You have no further evidence.
> *Answer.* Degree 0.7 stands out as uniquely reasonable.
>
> *(b)* You do have further evidence, namely some contingent atomic proposition F (e.g., the proposition that Finley, another goat, escapes).
> *Answer.* Still 0.7. Without any evidence that relates F to A, you have no grounds for any other choice. The chance of A gives robust grounds for believing A to degree 0.7.
>
> *(c)* You have instead evidence just that $A \leftrightarrow F$, for some contingent atomic proposition F (e.g., F says that Francis is brown and you learn that Ashley and Francis are identical twins, so $A \leftrightarrow F$).
> *Answer.* Still 0.7. With no other evidence bearing on F, learning that A and F have the same truth value doesn't tell you anything about A.

Landes et al. (2021) argue that such questionnaires serve to identify normal informal standards of what is reasonable. These are norms of reasonableness that are conformed to widely enough to be considered normal or standard requirements. For example: the claim that one is rationally required to believe to the same degree that a fair coin will land heads and that it will land tails; the claim that one is rationally permitted to bet on either outcome at even odds.

There is a difference between these informal nisowir and formal conditions that can be used explicate them. Lewis' Principal Principle, introduced in §4, can be considered a formal explication of the nisowir that are elicited by his questionnaire. The auxiliary admissibility conditions 1 and 2, also introduced in §4, explicate the nisowir elicited by scenarios (*b*) and (*c*) in the questionnaire of Landes et al. (2021) quoted above.

Something needs to be said here about what it is to explicate nisowir. This is because the term 'explication' was introduced by Carnap to apply only to concepts, not to nisowir:

> The task of *explication* consists in transforming a given more or less inexact concept into an exact one or, rather, in replacing the first by the second. (Carnap, 1950, p. 3.)

For example, the concept *temperature* explicates the comparative concept *warmer* (Carnap, 1950, p. 12).[10]

We can understand an explication of nisowir to be analogous to Carnap's notion of explication of a concept. The task of explication of nisowir consists in replacing the more or less inexact nisowir by an exact standard of what is reasonable. Note that such an explication often needs to subsume a whole class of similar nisowir. The Principal Principle, for example, replaces many inexact nisowir concerning reasonable degree of belief that can be elicited from Lewis' questionnaire and others. Thus an explication of nisowir often needs to generalise, but not overgeneralise, a class of related nisowir. An explication of nisowir also needs to satisfy some other desiderata. Carnap provides four requirements for an explication of a concept and these apply equally to an explication of nisowir: similarity to the explicanda, exactness, fruitfulness and simplicity (Carnap, 1950, §3).

An explication of nisowir may invoke Carnapian explications of concepts. For example, *PP* explicates nisowir and this explication presupposes that the concept of rational degree of belief is explicated by mathematical probability. (This move is appropriate here because the argument seeks to point out a consequence of the Bayesian framework. This move would not be appropriate in the context of Cox's theorem, which can be thought of as an argument in favour of a probabilistic explication of the concept of rational degree of belief.)

The possibility arises that certain nisowir might be explicated in different, mutually incompatible ways. Moreover, the current evidence may underdetermine which of these possible explications best balances the five desiderata introduced above—

[10]While Carnap gave the process of explication its name, he was not the first to discuss this process. For example, Matthew Young argues as follows:

> pleasure and pain, heat and cold, probability and improbability, virtue and vice, which are estimated by degrees, are not measurable. … Now to make quantities which consist of degrees, and therefore are not measurable, the subject of mathematical comparison, an arbitrary measure is assigned, by referring them to some measurable quantity to which they are related. Thus, in the graduation of the thermometer, an arbitrary measure is established for heat and cold, for the degrees of heat are referred to the expansion of the fluid contained in the thermometer, which is measurable, and to which heat is related. In the same manner, probability has no measure in itself; but an arbitrary measure is assigned to it, by referring it to the ratio of the number of chances by which the event may happen or fail; and thus it becomes the subject of mathematical calculation, in the same manner as the degrees of heat. (Young, 1800, pp. 80–81.)

i.e., the requirements that explications should appropriately generalise the nisowir, be similar to the nisowir, and be exact, fruitful and simple.

It may be, however, that one of these explications stands out as best in some particular context. For example, in the context of the standard Bayesian framework, the Principal Principle arguably stands out as being the most natural way to explicate the nisowir elicited by Lewis' questionnaire. Such an explication may not stand out as best in other contexts. For example, the Principal Principle would not be appropriate in the context of the framework of Dempster-Shafer belief functions, which does not presuppose that rational degree of belief can be explicated by conditional probability.

Suppose that some explication of nisowir stands out as best, by a long way, given the standard presuppositions of the context. We shall call such an explication a *canonical* explication of nisowir in that context.

§5.2. How nisowir can resolve MP/MT

An appeal to a canonical explication of nisowir can help with the MP/MT problem. If *PP* can be justified as a canonical explication of nisowir, and no such justification can be provided for $\neg PoI$, then this would favour the modus ponens over the modus tollens in the $PP \rightarrow PoI$ argument introduced in §4.

Such a line of reasoning might be motivated more fully as follows:

1. Those informal standards of what is reasonable that have become entrenched as nisowir have become so because they have been particularly conducive to our survival or to our achieving other goals.

2. Therefore, it is indeed reasonable to conform to such a norm. I.e., a nisowir is justified, in the absence of any evidence against its reasonableness.

3. The role of an exact normative theory such as Bayesianism is to explicate and unify many nisowir, and to resolve any inconsistencies between nisowir, in order to provide guidance in complex situations where the nisowir do not suffice on their own.

4. In the context of standard Bayesianism, *PP* is a canonical explication of nisowir.

5. Therefore, in the context of standard Bayesianism, *PP* is justified in virtue of the nisowir that *PP* explicates being justified.

6. That *PP* implies *PoI* is justified by a mathematical proof.

7. Therefore, *PoI* is justified by a valid argument (by modus ponens) from justified premisses, in the context of standard Bayesianism.

This line of reasoning, if successful, shifts the burden of proof onto the proponent of the modus tollens. The MT proponent cannot simply respond with an analogous line of reasoning, because $\neg PoI$ is not a canonical explication of nisowir. $\neg PoI$ says that one is not rationally required to be indifferent between an atomic proposition and its negation, in the absence of any evidence relevant to that proposition. While some do indeed endorse this claim, many take the opposite view. Thus this claim is hardly a normal informal standard of what is reasonable. The fact is that the Principle of Indifference is very contentious—much more so than the Principal Principle.

This line of reasoning can be generalised to mitigate the MP/MT problem in a broad range of scenarios. Most directly, if the second premiss θ of a MP is any formal statement about what is reasonable or rational, then we can ask if it is a canonical explication of nisowir, in the relevant context. If it is, we can appeal to an analogous line of reasoning. As we have seen, one can argue that while PP is a canonical explication of nisowir, $\neg PoI$ is not. In the case of Cox's argument, neither the conjunction of the conditions of Cox's theorem nor the negation of its conclusion can plausibly be construed as a canonical explication of nisowir. Thus while this line of reasoning shifts the burden of proof to Bayesians who reject PoI, it favours neither Cox's MP nor the anti-Cox MT. This suggests that, ceteris paribus, Cox's MP fails to provide a persuasive argument for probabilism, and its detractors' MT fails to provide a persuasive argument against Cox's common-sense conditions.

The above line of reasoning can also be generalised to the situation in which θ is not a formal statement but is instead an informal statement about what is reasonable. In that case, we can ask whether θ is a nisowir, rather than a canonical explication of nisowir. If so, we can invoke a version of the above line of reasoning, but omitting steps 3–5.

A further generalisation is possible: if θ is any other proposition, one can ask whether normal informal standards of what is reasonable favour believing θ, when one is pressed on the matter. This consideration helps neither side of the Putnam-Quine disagreement, because both metaphysical realism and the negation of ontological relativity go beyond the diktats of normal standards of what is reasonable. A similar point can be made about the pro-Trump / anti-Trump instance of MP/MT. However, this consideration is arguably applicable to the common-sense response to scepticism. Consider the common-sense modus tollens first. When I apparently hold out my hand in front of me, it would infringe normal informal standards of what is reasonable if I were to deny (or even, when pressed, withhold judgement on) the claim that I know that this is a human hand in front of me. Thus nisowir favour believing the second premiss of the common-sense MT. On the other hand, nisowir do not apparently favour believing the second premiss of the sceptical modus ponens, namely that I don't know that I'm not a brain in a vat. Questionnaires could be used to support these two claims about nisowir. Such questionnaires would then provide a public justification of the two claims that would shift the burden of proof to the sceptic.

To summarise: if (i) the second premiss of a philosophical modus ponens is a nisowir or a canonical explication of nisowir, or nisowir favour believing the second premiss when pressed, and (ii) the second premiss of the corresponding modus tollens does not satisfy this condition, then there are grounds for siding with the modus ponens, against the modus tollens.[11] Any public justification of (i) and (ii)—for example, a Lewis-style questionnaire—can, when taken together with a public justification for the first premiss, provide the means to persuade an interested third party of the conclusion of the modus ponens. On the other hand, if nisowir underpin the second premiss of the modus tollens but not the second premiss of

[11] It is crucial not to overlook condition (ii) here. For example, in order to maintain that the $PP \to PoI$ modus ponens is in a better position than the modus tollens, one needs not only to motivate the second premiss of the MP by appeal to nisowir but also to make a case that the second premiss of the MT cannot be so motivated.

the modus ponens, then suitable public justifications of analogues of (i) and (ii) will shift the burden of proof to the MP proponent.

§5.3. Discussion

It is important to emphasise that the appeal to nisowir is intended to be burden-shifting rather than decisive. When applied to the common-sense response to scepticism, for example, it does not amount to a refutation of scepticism. But a shift in the burden of proof is enough to ward off the challenge posed by MP/MT because it is enough to favour one side over the other. If the MP is justified by nisowir, but the MT is not, then it is not reasonable to respond to the MP by advocating the MT, in the absence of some suitable public justification of the MT that does not also apply to the MP. The philosophical argument in question is thus not a pantomeme.

It is also worth observing that the above motivation in terms of claims 1–7 appeals to certain empirical assertions. Claim 1, for example, is an empirical assertion; indeed, the origin of norms is an important question in sociology (Horne and Mollborn, 2020).[12] In addition, the question of whether a purported nisowir is indeed a nisowir is an empirical question. The questionnaires of Lewis (1980) and Landes et al. (2021) are intended to help us recognise nisowir. Where it is uncertain whether a purported nisowir is really a nisowir, a more structured survey may be required to settle the question; the methods of experimental philosophy can be of assistance in this regard. No attempt will be made at a sustained defence of these empirical claims here. Rather, the thesis of this section is that if certain empirical claims are true then the MP/MT problem can be mitigated.

In addition, the motivation of §5.2 is intended to be explanatory, rather than persuasive. It is intended to explain how an appeal to nisowir can resolve the MP/MT problem, rather than persuade a detractor that it solves the problem. This more limited ambition is appropriate because such a line of reasoning is itself a philosophical argument and thus prone to the MP/MT problem. To see this, note that the argument might be summarised as concluding that the MP/MT problem can be solved in certain situations (*MS*) on the grounds that nisowir have normative force (*NN*). This can be viewed as an argument by modus ponens, and opens the door to a response by modus tollens:

MP (pro solution)	MT (anti solution)
$\dfrac{NN \rightarrow MS \quad NN}{MS}$	$\dfrac{NN \rightarrow MS \quad \neg MS}{\neg NN}$

The pro-solution modus ponens is self-explanatory because, if sound, it explains why one should believe the contentious second premiss, and hence why one

[12] Claim 1 does not require that each individual nisowir confers evolutionary advantage; it is sufficient that the human capacity to generate such norms leads to evolutionary advantage (Clark, 1990), or indeed some other sort of advantage, so that, in general, nisowir are likely to lead to advantage. Moreover, it is enough that individual nisowir are usually only of heuristic value, reliable in a typical range of circumstances, but not all. Such a nisowir can be relied upon unless the circumstance is known to be exceptional.

One might be concerned that claim 1 merely provides pragmatic motivation for claim 2, when what is needed is an epistemic justification. However, the step from 1 to 2 can be made on epistemic grounds: one can argue that nisowir have become entrenched precisely because they lead to beliefs that are likely to be true; hence it is reasonable to conform to them.

should believe the conclusion. This is because normal informal standards of what is reasonable favour believing *NN*, i.e., that nisowir have normative force, when pressed to take some attitude towards *NN*. To see this, note for instance that if it is a nisowir that one ought to equivocate between a fair coin landing heads or tails, then normal standards favour believing—when pressed—that one ought to so equivocate. Hence if we grant that the MP is sound and thus that *NN* is true, one can see why one would be justified in believing *NN*. This helps to explain why one would be justified in believing the conclusion, *MS*.

Of course there is a circularity to this line of reasoning: the explanation of *NN* presumes its truth. Hence the ambition of the reasoning of §5.2 is limited to explanation rather than persuasion. That a particular explanation only succeeds in explaining if it is true is not in itself a problem—many of our best explanations have this characteristic.[13]

In this section, then, we have seen that an appeal to nisowir, and canonical explications of nisowir, can provide a burden-shifting resolution to the MP/MT problem. We have also seen how this strategy can be applied to resolve the debate around $PP \to POI$ in favour of the modus ponens and the debate about scepticism in favour of the modus tollens, should suitable public justifications be provided. (The focus here has been on developing a general strategy to address MP/MT, rather than on providing detailed public justifications to support claims about particular arguments.) I have also developed a line of reasoning that—if its empirical claims are correct—can explain why the nisowir strategy is successful. In the next section I put forward another potential response to the MP/MT problem that can be used in combination with an appeal to nisowir.

§6
Argument by interpretation

Recall our two examples in formal epistemology, introduced in §4. We have seen that the MP/MT problem can be mitigated in the case of the argument of Hawthorne et al. (2017) if we appeal to canonical explications of nisowir. However, this strategy does not help Cox's argument. The question thus arises as to how best to motivate probabilism, given that Cox's argument falls to MP/MT. This question is important here because the argument of Hawthorne et al. (2017) presupposes probabilism.

Accuracy arguments for probabilism provide an alternative to Cox's argument (Joyce, 1998; Predd et al., 2009; Pettigrew, 2016). However, these arguments are also susceptible to MP/MT, as we shall now see. Accuracy arguments for probabilism can usually be summarised as follows: rational degrees of belief minimise inaccuracy (***MI***), and an accuracy measure satisfies certain technical conditions including strict propriety (***SP***), so rational degrees of belief are probabilities (***BP***).

[13]One might think that this circularity precludes any grounds for favouring the pro-solution MP over the anti-solution MT. However, that conclusion would be too quick. The pro-solution modus ponens, I have argued, is at least self-explanatory. On the other hand, the anti-solution modus tollens is not even self-explanatory: the truth of $\neg MS$ would not explain why one would be justified in believing $\neg MS$. To the extent that being self-explanatory is a virtue of an argument, this virtue favours the pro-solution MP over the anti-solution MT. Hence, that the MP is explanatory counts somewhat in its favour, despite the inherent circularity.

Note that this reasoning is stable under the logical transformation discussed in §1, where, by taking the contrapositive of the first premiss, one can construe the pro-solution argument as a modus tollens and the anti-solution response as a modus ponens.

Strict propriety says that an accuracy function a should be such that each belief function *Bel* uniquely maximises its own expected accuracy,

$$\sum_{\omega\in\Omega} Bel(\omega)a(Bel,\omega),$$

where Ω is the set of possible worlds. The problem is that *SP* is very far from being a nisowir or an explication of nisowir. This opens the door to MP/MT:

MP (pro accuracy)	MT (anti accuracy)
$\dfrac{\begin{array}{r}(MI \wedge SP) \rightarrow BP \\ MI \wedge SP\end{array}}{BP}$	$\dfrac{\begin{array}{r}(MI \wedge SP) \rightarrow BP \\ \neg BP\end{array}}{\neg(MI \wedge SP)}$

Attempts to provide philosophical arguments for *SP* are prone to the same problem: they are not themselves grounded in nisowir (see, e.g., Campbell-Moore and Levinstein, 2021). Thus, accuracy arguments apparently fail to provide a viable alternative to Cox's argument.

Perhaps the classic argument for probabilism is the Dutch book argument. This can be formulated as follows: degrees of belief are betting quotients (*BQ*) and rational betting quotients avoid sure loss (*AL*), so rational degrees of belief are probabilities (*BP*) (see, e.g., Paris, 1994, Chapter 3; Gillies, 2000, Chapter 4). A betting quotient is a value at which one would be prepared to bet for or against a given proposition: x is a betting quotient for proposition θ if one would consider xS a fair price to pay to receive S in return if θ turns out to be true, for an unknown stake S that may be positive or negative. The question is whether MP/MT poses a problem for the Dutch book argument:

MP (Dutch book)	MT (anti Dutch book)
$\dfrac{\begin{array}{r}(BQ \wedge AL) \rightarrow BP \\ BQ \wedge AL\end{array}}{BP}$	$\dfrac{\begin{array}{r}(BQ \wedge AL) \rightarrow BP \\ \neg BP\end{array}}{\neg(BQ \wedge AL)}$

While normal standards of what is reasonable might favour believing that rational betting quotients avoid sure loss (*AL*), the same cannot be said for the claim that degrees of belief are betting quotients, *BQ*. *BQ* is an interpretation or explication of one's degree of belief in θ as the betting rate that one would consider fair, whether betting for or against θ. Hence the Dutch book MP can be viewed as an 'argument by interpretation' (Williamson, 2010, §3.1): a key premiss proposition, *BQ*, provides an exact interpretation of degree of belief, and the viability of the argument hinges on this interpretation.

As Ramsey notes,

> the degree of a belief ... has no precise meaning unless we specify more exactly how it is to be measured. (Ramsey, 1926, p. 167.)
>
> The old-established way of measuring a person's belief is to propose a bet, and see what are the lowest odds which he will accept. (Ramsey, 1926, p. 172.)

The latter quote suggests that the betting-quotient interpretation of degree of belief might be a canonical explication of the concept of degree of belief—i.e., an explication that stands out as best, by a long shot, given the standard presuppositions of the context.

If so, this provides a further means to address the MP/MT problem: an appeal to canonical explications of concepts, in addition to nisowir and canonical explications of nisowir. The reasoning in favour of the Dutch book MP would thus be as follows: the first premiss is justified by a mathematical proof; the first conjunct BQ of the second premiss is justified in virtue of being a canonical explication of a concept; the second conjunct AL is justified by nisowir. If successful, this shifts the burden of proof to the proponent of the anti Dutch book MT. Such a proponent must either provide grounds to reject the mathematical proof, the canonical explication of the concept of degree of belief, or the nisowir behind the claim that betting quotients that incur sure loss are irrational.

If the MT proponent were to reject the interpretation of degree of belief to which the MP appeals, the worry would arise that the MP proponent and the MT proponent are simply talking past each other. Thus the burden is on the MT proponent to provide grounds for denying that betting quotients offer a canonical explication of the concept of degree of belief. The natural way to do this would be to provide an alternative explication of the concept and argue that the betting-quotient explication does not stand out as superior to this alternative explication.

For example, the MT proponent might admit that the betting-quotient interpretation is exact, fruitful and simple, but point out that we do not, in practice, bet at the same rate for or against a proposition (see, e.g., Walley, 1991, p. 3), raising this as a concern for the claim that the betting-quotient explication is sufficiently similar to the explicandum. The MT proponent might argue that this alternative dual-rate explication is better than the single-rate betting-quotient explication of degree of belief.

However, the advocate of the betting-quotient explication can resist this conclusion by observing that, when one allows different buy and sell rates, the Dutch book argument yields imprecise probability instead of standard precise probability, and that this leads to certain disadvantages. Imprecise probability is certainly less simple (see, e.g., Augustin et al., 2014; Troffaes and de Cooman, 2014) and arguably less fruitful than precise probability, because it is a strictly weaker framework. Moreover, even if different rates for buying and selling bets are more realistic, that does not on its own imply that this alternative betting set-up is any closer to the explicandum than the single-rate betting-quotient approach. Indeed, while both precise and imprecise probabilists would take single-rate betting quotients to be an obvious indicator of strength of belief, it is much more doubtful that what is measured by two different rates for buying and selling is strength of belief, as opposed to, say, risk aversion.

In sum, then, the proponent of the Dutch book argument for probabilism can justify the claim that degrees of belief are betting quotients by invoking it as a canonical explication of the concept of degree of belief. The Dutch book argument can be thought of as an argument by interpretation, and this argumentative strategy provides a further weapon in the arsenal against MP/MT.

§7
Conclusion

That one philosopher's modus ponens is another's modus tollens poses a serious challenge to philosophical practice. In order to decide between the modus ponens and the modus tollens one needs public justifications that can favour one argument over the other. Otherwise, philosophical arguments are mere pantomemes.

Mathematical proof is one kind of public justification, but it is generally not applicable to all the premisses of a philosophical argument, even in areas such as formal epistemology. There are other kinds of public justification, however: evidence of normal informal standards of what is reasonable (nisowir), canonical explications of nisowir, and canonical explications of concepts can also be used to justify philosophical premisses. The Dutch book argument for probabilism is interesting because it involves all three of these tools.[14]

This toolkit requires a shift in philosophical methodology. It is an empirical question whether a claim about what is reasonable is a nisowir. Answering such a question may require an appeal to questionnaires, such as the one provided by David Lewis, or the more systematic surveys of experimental philosophy. In addition, determining whether an explication is canonical requires careful weighing of desiderata. As Carnap noted,

> The question whether the solution is right or wrong makes no good sense because there is no clear-cut answer. The question should rather be whether the proposed solution is satisfactory. (Carnap, 1950, p. 4.)

To give a recent example, some might argue that Pearl's mathematisation of causality in terms of conditional probabilistic independence qualifies as a canonical explication of causality (Pearl, 1988, 2000). It is exact, fruitful and simple. Is it sufficiently similar to the explicandum? On the one hand, some have argued that it admits counterexamples (see, e.g., McKim and Turner, 1997; Williamson, 2005), and we might also question whether it can successfully accommodate the rich epistemology of causality, which arguably seeks evidence of mechanisms in addition to evidence of correlation (Russo and Williamson, 2007). On the other hand, explication is a transformative process: it is the replacement of something informal by something exact, and this process can tolerate some significant discrepancies between explicandum and explicatum. When probability was axiomatized, for example, the informal concept, which was not universally taken to be additive (Bernoulli, 1713), was replaced by an additive concept (Kolmogorov, 1933). This kind of additive mathematisation of probability quickly became entrenched in mathematics and statistics as the canonical explication of the informal concept. The question thus arises whether the informal concept of causality might eventually be replaced by Pearl's mathematisation despite the infelicity of the explication. Time will tell; in this case we are apparently not yet in a position to make a judgement. In general, it can be very difficult to evaluate explications without some benefit of hindsight.

A philosophical methodology that has a central place for nisowir and explication thus requires considerations rather alien to the usual a priori conceptual analysis and metaphysical theorising that currently characterise analytic philosophy. But some such transformation is required to avoid the impasse of MP/MT.

[14] There is no claim here that these tools are exhaustive, i.e., that they provide the only means of addressing the MP/MT problem. MP/MT poses a challenge that must be met, however. If there are other tools, it is important to identify them and explain how they work.

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