London Mathematical Society Research School

Orthogonal Polynomials and Special Functions

26th-30th June, 2017

School of Mathematics, Statistics & Actuarial Science Sibson Building, Parkwood Road University of Kent Canterbury, CT2 7FS, UK

Organisers

Peter Clarkson (University of Kent, Canterbury, UK) Ana Loureiro (University of Kent, Canterbury, UK)

Information Booklet







Welcome to the London Mathematical Society Research School **Orthogonal Polynomials and Special Functions**. The Research School is also an activity of the SIAM Activity Group on Orthogonal Polynomials and Special Functions; this is the seventh School on Orthogonal Polynomials and Special Functions. The activity group promotes basic research in orthogonal polynomials and special functions; further the application of this subject in other parts of mathematics, and in science and industry. It also encourages and supports the exchange of information, ideas, and techniques between workers in this field and other mathematicians and scientists.

The Research School consists of three courses by eminent Mathematicians in the field of orthogonal polynomials and special functions, each with five lectures and two tutorial sessions, supplemented by three guest lectures.

We thank the London Mathematical Society (LMS) for their generous funding of the Research School. We are grateful to Elizabeth Fisher (LMS) and Louisa Harvey (University of Kent) for their assistance in the organisation of the Research School.

We hope that you'll enjoy the Research School, learn much and enhance your appreciation of the wonderful world of orthogonal polynomials and special functions.

Peter Clarkson and Ana Loureiro

Course Lecturers

Kerstin Jordaan Nalini Joshi Walter Van Assche University of South Africa, Pretoria, South Africa University of Sydney, Australia University of Leuven, Belgium

Guest Lecturers

Andrew Hone Andrei Martínez-Finkelshtein Adri Olde Daalhuis University of Kent, Canterbury, UK University of Almería, Spain University of Edinburgh, UK

Tutorial Assistants

Alfredo Deaño	University of Kent, Canterbury, UK
Andrew Hone	University of Kent, Canterbury, UK
Ana Loureiro	University of Kent, Canterbury, UK

Local Organising Committee

Claire Carter Peter Clarkson Alfredo Deaño Ana Loureiro Elizabeth Mansfield John Pearson Ian Wood

	Monday 26 June	Tuesday 27 June	Wednesday 28 June	Thursday 29 June	Friday 30 June
08:00-09:00	Registration & Opening				
09:00-10:00	Lecture Jordaan 1	Lecture Van Assche 2	Lecture Van Assche 4	Lecture Joshi 5	Lecture Jordaan 5
10:00-11:00	Lecture Joshi 1	Lecture Jordaan 2	Lecture Joshi 4	Lecture Jordaan 4	Lecture Van Assche 5
11:00-11:30	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
11:30-12:30	Tutorial Jordaan 1	Tutorial Joshi 1	Lecture Jordaan 3	Tutorial Van Assche 2	Q&A session
12:30-14:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:00-15:00	Lecture Van Assche 1	Lecture Joshi 3	Tutorial Joshi 2	Aurora event (Joshi)	Guest Lecture Martínez-Finkelshtein
15:00-16:00	Tutorial Van Assche 1	Lecture Van Assche 3		Tutorial Jordaan 2	
16:00-16:30	Coffee break	Coffee break		Coffee break	
16:30–17:30	Lecture Joshi 2	Guest Lecture Hone		Guest Lecture Olde Daalhuis	
	Drinks Reception			School Dinner	

OPSF Research School Timetable



Kerstin Jordaan



Nalini Joshi



Walter Van Assche

Lecture Course 1: "Properties of orthogonal polynomials" Kerstin Jordaan (University of South Africa, Pretoria, South Africa)

Abstract. In these lectures, an introduction will be given to the theory of orthogonal polynomials. We will discuss basic concepts and classical properties of orthogonal polynomials within the context of applications. The lectures aim to show, by means of accessible examples, that interesting research problems arise from asking questions about the characteristic properties satisfied by classical orthogonal polynomials and their extensions or generalisations. Topics to be covered include the three term recurrence relation and recurrence coefficients, the spectral theorem, Hankel determinants, Jacobi matrices, Rodrigues type formulae, Padé approximants to hypergeometric functions, the Askey scheme of hypergeometric polynomials, contiguous relations, properties of zeros of orthogonal polynomials, Markov's monotonicity theorem, Sturm's convexity theorem, quasi-orthogonal polynomials, linear combinations of orthogonal polynomials.

Lecture Course 2:

"Discrete Painlevé Equations" Nalini Joshi (University of Sydney, Australia)

Abstract. In these lectures, an introduction will be given to discrete Painlevé equations, which are discrete analogues of the well known Painlevé equations. The solutions of the Painlevé equations appear as universal nonlinear mathematical models in many applications. The study of their discrete versions stands at the leading edge of this field and has created an extraordinarily rich vein of new ideas and methods over the past 20 years. It is now clear that these discrete versions will be useful and applicable, yet their properties are not well known to most mathematicians and mathematical scientists. This course of lectures will provide an introduction to nonlinear discrete equations, an overview of properties of discrete Painlevé equations, a toolbox to describe their phase space in terms of singularities and geometry, and how this may be mined to provide asymptotic behaviours of solutions. The lectures will provide training and information for students, who may encounter the nonlinear discrete equations, and in particular, the discrete Painlevé equations, as mathematical or physical models in the course of their research.

Lecture Course 3:

"Multiple Orthogonal Polynomials" Walter Van Assche (University of Leuven, Belgium)

Abstract. Multiple orthogonal polynomials are polynomials of one variable that satisfy orthogonality conditions with respect to r > 1 measures. They appeared as denominators of Hermite-Padé approximants to several functions in the 19th century and for that reason they are also known as Hermite-Padé polynomials. Other names used in the literature are polyorthogonal polynomials and d-orthogonal polynomials, which are basically the type II multiple orthogonal polynomials near the diagonal (the so-called stepline) and d corresponds to the number of measures (which we denote by r). Multiple orthogonal polynomials were studied in the 20th century by Mahler who obtained many properties, in particular the existence and normality (perfect systems). Analytic properties, in particular the asymptotic behaviour of the zeros, were obtained at the end of the 20th century. In the 21st century multiple orthogonal polynomials started to appear in various random matrix models, non-intersecting random paths, and other determinantal processes. In particular the average characteristic polynomial of a random matrix often turns out to be a multiple orthogonal polynomial so that the eigenvalues of a random matrix are expected to behave like the zeros of a multiple orthogonal polynomial. Multiple orthogonal polynomials also are useful to describe certain discrete integrable processes. In these lectures an introduction and a discussion of the most important families of multiple orthogonal polynomials will be given together with their known properties and applications.

Guest Lecture 1:

"Continued fractions and nonlinear recurrences" Andrew Hone (University of Kent, Canterbury, UK)

Abstract. Three-term linear recurrences are a central part of the theory of continued fractions, in description of convergents, as well as being a feature of orthogonal polynomials. This talk will consider some examples of nonlinear recurrences associated with continued fractions. In the setting of the real numbers, some new examples of transcendental numbers with an explicit continued fraction expansion will be provided. The case of function fields associated with algebraic curves will also be mentioned, pointing out the connection with certain discrete integrable systems, Hankel determinants and orthogonal polynomials.

Guest Lecture 2:

"Vector equilibrium and asymptotics of zeros of multiple orthogonal polynomials" Andrei Martínez-Finkelshtein (University of Almería, Spain)

Abstract. It is known since the end of the 20th century that the analytic properties, in particular the asymptotic behaviour of the zeros, of families of polynomials that satisfy orthogonality conditions with respect to several measures (multiple orthogonal polynomials) have an electrostatic description in terms of vector-valued measures. On the real line these measures typically provide a global minimum of an associated energy functional, but when we deal with non-hermitian orthogonality, a more general notion of equilibrium must be considered. The first part of the lecture contains a general introduction to these notions, while in the second one we discuss the case of multiple orthogonal polynomials with respect to a cubic weight, where the integration goes along non-homotopic paths on the plane. For their asymptotic description we need to analyze saddle points of an energy functional in which the mutual interaction comprises both attracting and repelling forces. The resulting measures can be characterized by a cubic algebraic equation (spectral curve) whose solutions are appropriate combinations of the Cauchy transform of its components. In particular, these measures are supported on a finite number of analytic arcs that are trajectories of a quadratic differential globally defined on a three-sheeted Riemann surface. The complete description of the so-called critical graph for such a differential (and its dynamics as a function of the parameters of the problem) is the key ingredient of the asymptotic analysis of the multiple orthogonal polynomials.

This is talk is partially based on the joint work with Guilherme L.F. Silva (University of Michigan, Ann Arbor, USA).

Guest Lecture 3:

"Exponential Asymptotics and Resurgence" Adri Olde Daalhuis (University of Edinburgh, UK)

Abstract. Exponential asymptototics has been a very active area of research in the last 3 decades. It started with fundamental work by Berry, Ecalle and Kruskal. Small exponentials are usually responsible for the divergence of asymptotic series. Resurgence enables the divergent tails to be decoded to yield these exponentials. Including these small exponentials leads to exponentially-improved asymptotics at several levels: hyperasymptotics. Via resurgence we are now able to compute the so-called connection coefficients, or Stokes multipliers. I will demonstrate the latest progress of exponential asymptotics for ordinary differential equations and partial differential equations.

These new techniques give us also better representations for the remainders in the asymptotic expansions and this has led recently to much sharper error bounds for the asymptotic expansions of many special functions.

The smoothing of the Stokes phenomenon via an error function was introduced by Berry in 1989. I will discuss the higher order Stokes phenomenon and its smoothing via combinations of error functions.

Information

Registration

Registration will take place 08:00–09:00 on Monday 3rd July in the Atrium in the Sibson building, where the Research School is taking place.

Venue

The Research School take place at Sibson Building (C2 on the campus map): Sibson Building, Parkwood Road University of Kent Canterbury, CT2 7FS

Floor plans for the ground and first floors of the Sibson building



Accommodation

Accommodation is located in **Turing College** (N7 on the campus map).

- Check-in is from 14:00 on your day of arrival and check-out is by 10:00 on your day of departure. Please visit Turing Reception (N7 on the campus map) upon your arrival to collect your room keys.
- Turing Reception will be open from 08:00 to 22:00 daily. If you are due to check-in to your accommodation after reception has closed you will need to contact Campus Security upon your arrival who will open reception. An internal phone is available outside reception to dial through to our 24hr security team.
- Bedrooms are serviced daily and provided with towels, linen and complimentary toiletries. Tea and coffee making facilities are also provided.
- Delegates are asked to familiarise themselves with the University's evacuation procedures, which are displayed on the back of the door in each bedroom.
- Breakfast is provided in Hut 8, Turing College between 07:45 and 09:30 and you will be required to show your room key to prove residency.
- An evening meal will be served in Rutherford Dining Hall from 18:30–19:30, on Sunday 25th June. Evening meals will be served in Hut 8, Turing College from 18:30–19:30, on Monday 26th-Wednesday 28th June.

Lectures & Tutorials

The lectures will take place in Sibson Seminar Room 6, on the first floor of the Sibson building. Tutorials will take place in Sibson Lecture Theatre 2, on the first floor of the Sibson building.

Coffee Breaks & Lunch

Coffee breaks will take place on the first floor of Sibson building throughout the Research School. Lunch will be served in the Sibson Atrium throughout the Research School.

Evening meals

An evening meal will be served in Rutherford Dining Hall from 18:30–19:30, on Sunday 25th June. Evening meals will be served in Hut 8, Turing College from 18:30–19:30, on Monday 26th-Wednesday 28th June.

Welcome Reception – Monday 26th June

On Monday 26th June there will be a welcome reception in the Sibson Atrium from 17:30–18:30.

Aurora @Kent Event - Thursday 29th June

On Thursday 29th June there will be the Aurora @Kent Event *Making waves: an interview with Professor Nalini Joshi*, 14:00-15:00, in Sibson Lecture Theatre 3, on the first floor of the Sibson building.

Research School Dinner – Thursday 29th June

The Research School dinner will take place on Thursday 6th July in the Darwin Conference Suite, which is on the ground floor of Darwin College, University of Kent. Pre-dinner drinks will be served from 19:30 with dinner at 20:00.

Sibson Café

The Sibson Café is located on the ground floor of the Sibson building, which is open 09:00–15:00 during the Research School. A range of hot and cold drinks, sandwiches, salads and snacks are available.





The Sibson Building

Internet Access

Wireless zones are available in teaching and social areas on campus. It is a condition of connecting to the internet using the Kent network that all laptops should be running current anti-virus software and the latest security updates.

Use the University of Kent WiFi service **eduroam** if you are from an **eduroam** institution. Delegates are not from an **eduroam** institution will be supplied with a computer log-in in order to access the internet during their stay. Log-ins and instructions (for the University network) will be supplied during registration. Alternatively, anyone can pick up WiFi Guest, free public WiFi provided by The Cloud, on campus.

How to get on visitor WiFi:

- find WiFi Guest in your list of available WiFi networks and select it;
- log in, or register if you're a first time user, to gain internet access.

Smoking

The University operates a non-smoking policy in all areas.

Messages

Between the hours of 09:00 and 17:00 (BST) messages for delegates can be left with the Conference team on +44 (0)1227 828000 or the SMSAS General Office +44 (0)1227 827181. Outside of these hours urgent messages only can be left with Campus Security on +44 (0)1227 823300

Campus facilities

Set in over 450 acres of beautiful parkland overlooking the historical city of Canterbury, the University of Kent campus provides a perfect location for conferences and events guests. We have a wide range of award-winning restaurants and bars, a cinema, theatre, shops, banking, as well as a gym and outdoor sports facilities.

Historic and Cosmopolitan Canterbury

The University of Kent campus is just a 20 minute walk or short bus journey into the city. Canterbury has something for everyone to enjoy – whether you want to take a boat ride on the River Stour, shop 'til you drop at big name stores and independent boutiques or sample some of the local culinary delights in award-winning restaurants and bars.

Tourist Information about Canterbury can be downloaded here.

A map of Canterbury can be downloaded here.

A map of Canterbury City Centre can be downloaded here.

A guide to top attractions in Canterbury, including a map, can be downloaded here.

Health/Medical Insurance

Delegates are advised to ensure that they have adequate medical insurance to cover the period of their stay in the United Kingdom.

Electrical Voltage

United Kingdom standard voltage is 240v, with square three-pin plugs — you may need to have an adaptor with you.



Participants

Atef Alaya

Gerardo Ariznabarreta **Dimitrios Askitis** Ahmad Barhoumi **Árpád Baricz** Kiran Kumar Behera **Geoffroy Bergeron** Oksana Bihun Éric-Olivier Bossé **Caroline Brett** Jian Cao Andrew Celsus Yoon Choun Sourav Das Abel Díaz González Ellen Dowie Alexander Dyachenko Julien Gaboriaud Juan Carlos García Ardila María Ángeles García-Ferrero Peter Gill Jean Carlo Guella Jesse Hayhow Wei Ying Hu Abey Sherif Kelil Rostyslav Kozhan Jean-Michel Lemay Helder Lima Jordan Makwana Juan Francisco Mañas Mañas Misael Marriaga Houry Melkonian Sheila Menchavez Proscovia Nabasinga Miria Sengul Nalci Tumer Hasen Ozturk Abdelsadek Saib Luis Sarmento Stella Schindler Alisa Sheinkman Mikhail Tyaglov **Tanay Wakhare** Yian Yao Yanely Zaldivar Gerpe Michele Zadra Weiwei Zhang

University of Gabès, Tunisia & Umm Al-Qura University, Makkah, Saudi Arabia Universidad Complutense de Madrid, Spain University of Copenhagen, Denmark Indiana University-Purdue University Indianapolis, USA Babeş-Bolyai University, Cluj-Napoca, Romania & Óbuda University, Budapest, Hungary Indian Institute of Technology Roorkee, India University of Montreal, Canada University of Colorado, Colorado Springs, USA Université de Montréal, Canada Lancaster University, UK Hangzhou Normal University, China University of Cambridge, UK University of Kent, Canterbury, UK Indian Institute of Technology Roorkee, India Universidad Carlos III de Madrid, Spain University of Kent, Canterbury, UK Technische Universität Berlin, Germany Université de Montréal, Canada Universidad Carlos III de Madrid, Spain ICMAT, Madrid, Spain Australian National University, Australia Universidade de São Paulo, Brazil Queen Mary University of London, UK City University of Hong Kong, China University of Pretoria, South Africa Uppsala University, Sweden Université de Montréal. Canada University of Kent, Canterbury, UK University of Kent, Canterbury, UK Universidad de Almería, Spain Universidad Carlos III de Madrid, Spain Heriot-Watt University, Edinburgh, UK Mindanao State University, Philippines UTAMU, Uganda Izmir Institute of Technology, Turkey University of Reading, UK University of Tebessa, Algeria

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