OPSFA14

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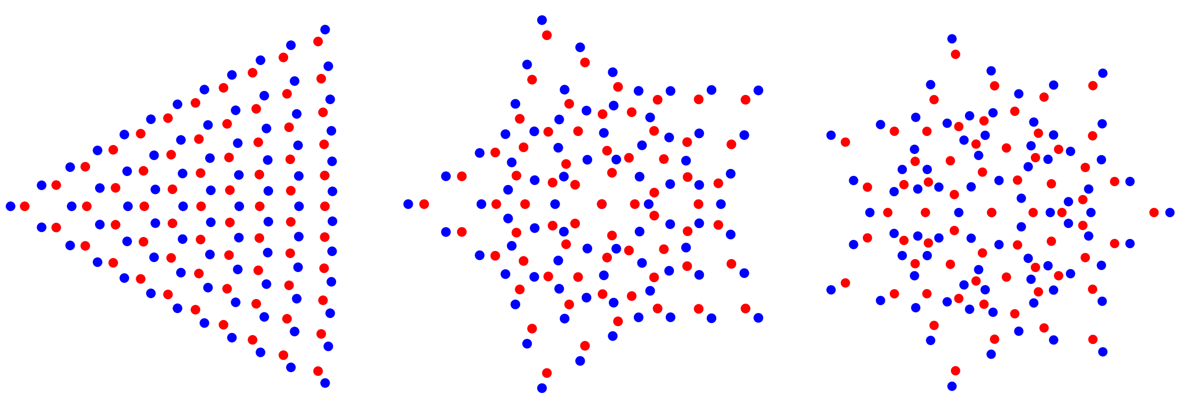
Orthogonal Polynomials, Special Functions and Applications

3rd–7th July 2017

School of Mathematics, Statistics & Actuarial Science
Sibson Building, Parkwood Road, University of Kent
Canterbury, CT2 7FS, UK

Abstracts



University of Kent

Plenary Talks

To infinity and back (a bit)

Jonathan Breuer (Hebrew University of Jerusalem, Israel)

10:00–11:00 Wednesday 5th July Sibson LR3

Let H be a self-adjoint operator defined on an infinite dimensional Hilbert space. Given some spectral information about H, such as the continuity of its spectral measure, what can be said about the asymptotic spectral properties of its finite dimensional approximations? This is a natural (and general) question, and can be used to frame many specific problems such as the asymptotics of zeros of orthogonal polynomials, or eigenvalues of random matrices. We shall discuss some old and new results in the context of this general framework and present various open problems.

Koornwinder polynomials at $q = t$

Sylvie Corteel (CNRS, Paris, France)

10:00–11:00 Frday 7th July Sibson LR3

In this talk, I will explain how to build Koornwinder polynomials at $q = t$ from moments of Askey-Wilson polynomials. I will use the classical combinatorial theory of Viennot for orthogonal polynomials and their moments. An extension of this theory allows to build multivariate orthogonal polynomials. The key step for this construction are a Cauchy identity for Koornwinder polynomials due to Mimachi and a Jacobi-Trudi formula for the 9th variation of Schur functions due to Nakagawa, Noumi, Shirakawa and Yamada. This is joint work with Olya Mandelshtam and Lauren Williams.

Exceptional orthogonal polynomials

David Gómez-Ullate (ICMAT and Universidad Complutense de Madrid, Spain)

09:00–10:00 Tuesday 4th July Sibson LR3

Exceptional orthogonal polynomials came as a little surprise to the community a few years ago, and now we are beginning to understand large parts of the theory and how they fit within the existing framework. They are complete families of orthogonal polynomials that arise as eigenfunctions of a Sturm-Liouville problem, and generalise the classical families of Hermite, Laguerre and Jacobi by allowing a finite number of gaps in their degree sequence. Despite these "missing" degrees, the remaining polynomials still span a complete basis of a weighted L^2 space, and the orthogonality weight is a rational modification of a classical weight. In this talk we will review the main results in the theory of exceptional orthogonal polynomials (classification, position of their zeros, recurrence relations, etc.), with emphasis on the similarities and differences with respect to their classical counterparts. We will also discuss some of their applications in mathematical physics, ranging from exact solutions to Schrödinger's equation in Quantum Mechanics to rational solutions of nonlinear integrable equations of Painlevé type.

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Computing symmetric cubatures: A moment matrix approach.

Evelyne Hubert (INRIA, Sophia Antipolis, France)

10:00–11:00 Thursday 6th July Sibson LR3

A quadrature is an approximation of the definite integral of a function by a weighted sum of function values at specified points, or nodes, within the domain of integration. Gaussian quadratures are constructed to yield exact results for any polynomials of degree $2r-1$ or less by a suitable choice of r nodes and weights. Cubature is a generalisation of quadrature in higher dimension. Constructing a cubature amounts to find a linear form $\Lambda : \mathbb{R}[x] \to \mathbb{R}$, $p \mapsto \sum_{j=1}^{r} a_j\, p(\xi_j)$ from the knowledge of its restriction to $\mathbb{R}[x]_{\leq d}$. The unknowns to be determined are the weights a_j and the nodes ξ_j.

An approach based on moment matrices was proposed in [2, 4, 6]. We give a basis-free version in terms of the Hankel operator $\mathscr{H}$ associated to Λ. The existence of a cubature of degree d with r nodes boils down to conditions of ranks and positive semi-definiteness on $\mathscr{H}$. We then recognise the nodes as the solutions of a generalised eigenvalue problem.

Standard domains of integration are symmetric under the action of a finite group. It is natural to look for cubatures that respect this symmetry [3, 5]. They are exact for all anti-symmetric functions beyond the degree of the cubature. Introducing adapted bases obtained from representation theory, the symmetry constraint allows to block diagonalise the Hankel operator $\mathscr{H}$. The size of the blocks is explicitly related to the orbit types of the nodes. From the computational point of view, we then deal with smaller-sized matrices both for securing the existence of the cubature and computing the nodes.

Joint work with Mathieu Collowald, Université Côte d'Azur & Inria Méditerranée [1].

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From Fourier's sombrero to Chebyshev's almond: approximation on the real line

Arieh Iserles (University of Cambridge, UK)

10:00–11:00 Monday 3rd July Sibson LR3

Motivated by the computation of quantum problems, we explore fast approximation of functions on the real line, in particular of wave packets – by "fast" we mean both spectral speed of convergence and derivation of the first n expansion coefficients in $\mathscr{O}(n \log n)$ operations. We explore four candidates: Hermite polynomials, Hermite functions, stretched Fourier series and stretched Chebyshev series, describe some unexpected phenomena and determine the surprising winner.

Monodromy dependence and connection formulae for isomonodromic tau functions

Alexander Its (Indiana University-Purdue University, Indianapolis, USA)

09:00–10:00 Thursday 6th July Sibson LR3

We discuss an extension of the Jimbo-Miwa-Ueno differential 1-form to a form closed on the full space of extended monodromy data of systems of linear ordinary differential equation with rational coefficients. This extension is based on the results of M. Bertola generalising a previous construction by B. Malgrange. We show how this 1-form can be used to solve a long-standing problem of evaluation of the connection formulae for the generic isomonodromic tau functions which would include an explicit computation of the relevant constant factors. We explain how this scheme works by calculating the connection constants for generic Painlevé VI (PVI) and Painlevé III (PIII) tau functions. The result proves the conjectural formulae for these constants earlier proposed by N. Iorgov, O. Lisovyy and Y. Tykhyy (PVI) and by O. Lisovyy, Y. Tykhyy and the speaker (PIII) with the help of the recently discovered connection of the Painlevé tau-functions with the Virasoro conformal blocks. The conformal block approach will be also outlined. The talk is based on the joint works with O. Lisovyy, Y. Tykhyy and A. Prokhorov.

Universality for conditional measures of the sine point process

Arno Kuijlaars (KU Leuven, Belgium)

09:00–10:00 Frday 7th July Sibson LR3

The sine process is a random point process that is obtained as a limit from the eigenvalues of many random matrices as the size tends to infinity. This phenomenon is called universality in random matrix theory, and it also holds for many orthogonal polynomial ensembles.

In this talk I want to emphasise another connection of the sine point process with orthogonal polynomials. It comes from a surprising property called number rigidity in the sense of Ghosh and Peres. This means that for almost all configurations, the number of points in an interval $[-R,R]$ is determined exactly by the points outside the interval. The conditional measures is the joint distribution of the points in $[-R,R]$ given the points outside. Bufetov showed that these are orthogonal polynomial ensembles with a weight that comes from the points outside $[-R,R]$.

I will report on recent work with Erwin Mina-Diaz (arXiv:1703.02349) where we prove a universality result for these orthogonal polynomial ensembles that in particular implies that the correlation kernel of the orthogonal polynomial ensemble tends to the sine kernel as R tends to infinity. This answers a question posed by Alexander Bufetov.

Painlevé equations, q-Askey scheme and colliding holes on Riemann surfaces

Marta Mazzocco (Loughborough University, UK)

09:00–10:00 Wednesday 5th July Sibson LR3

The monodromy manifold of the sixth Painlevé differential equation is a Poisson algebra that admits a natural quantisation to the Askey-Wilson algebra, encoding the symmetries of the Askey-Wilson polynomials. On the other side, this same monodromy manifold is the moduli space of monodromy representations of a Riemann sphere with 4 boundary components. In this talk we will show that by merging boundary components on this Riemann sphere, other Painlevé equations emerge in such a way that the quantization of their monodromy manifolds encode the symmetries of other families of basic hypergeometric polynomials belonging to the q-Askey scheme.

Rational solutions of Painlevé equations

Peter Miller (University of Michigan, Ann Arbor, USA)

10:00–11:00 Tuesday 4th July Sibson LR3

Most solutions of the famous Painlevé differential equations are highly transcendental, yet all but the Painlevé-I equation admit particular solutions that are elementary rational functions. These particular solutions are important in diverse applications, including the description of equilibrium patterns of fluid vortices, universal phenomena in nonlinear wave theory, electrochemistry, and string theory. This talk will illustrate some features of these particular solutions, describe how they may be obtained by iterated Bäcklund transformations or via the solution of appropriate Riemann-Hilbert problems, and focus attention on recent and ongoing efforts by several researchers to study families of rational Painlevé solutions in the asymptotic limit of large degree.

Integral representations for multivariable Bessel functions and beta distributions

Margit Rösler (University of Paderborn, Germany)

11:30–12:30 Monday 3rd July Sibson LR3

There exist various interesting classes of multivariable Bessel functions, such as Bessel functions of matrix argument which are important in multivariate statistics, or the Bessel functions associated with root systems in Dunkl theory. Such Bessel functions occur for certain discrete parameters in various contexts of radial analysis on Euclidean spaces.

In this talk, we shall first review some basics on Bessel functions of matrix argument and Dunkl-type Bessel functions, and explain their interrelation. We shall then focus on integral representations for multivariable Bessel functions which generalise the classical Sonine integral for the one-variable Bessel function $j_\alpha(z) = {}_0F_1(\alpha+1; -z^2/4)$,

$$j_{\alpha+\beta}(z) = \frac{2\Gamma(\alpha+\beta+1)}{\Gamma(\alpha+1)\Gamma(\beta)} \int_0^1 j_\alpha(zt)\, t^{2\alpha+1}(1-t^2)^{\beta-1}\,\mathrm{d}t, \qquad \alpha > -1, \quad \beta > 0.$$

For Bessel functions of matrix argument there are analogous representations, going back already to Herz, and for certain Bessel functions associated with root systems of type B there are similar results by Macdonald. In these known cases, however, the range of indices is restricted. Similar to the theory of Gindikin for Riesz measures, we shall extend these integral representations to larger index sets by means of tempered distributions, und study under which conditions these distributions are actually given by positive measures. There turn out to be gaps in the admissible range of indices which are determined by the so-called Wallach set.

As a consequence, we shall obtain examples where the Dunkl intertwining operator between Dunkl operators associated with multiplicities $k \geq 0$ and $k' \geq k$ is not positive, which disproves a long-standing conjecture.

The talk is based on joint work with Michael Voit, Dortmund.

Every moment brings a treasure: random matrix theory and moments of the Riemann zeta function

Nina Snaith (University of Bristol, UK)

11:30–12:30 Tuesday 4th July Sibson LR3

There has been very convincing evidence since the 1970s that the positions of zeros of the Riemann zeta function show the same statistical distribution (in the appropriate limit) as eigenvalues of random matrices. This talk will review how this connection was exploited in order to gain insight into average values of the zeta function and its derivative.

Non-smooth waves and Lax integrability; the playground of approximation theory and the theory of distributions

Jacek Szmigielski (University of Saskatchewan, Saskatoon, Canada)

11:30–12:30 Wednesday 5th July Sibson LR3

In the last two decades several models have been proposed to describe non-smooth waves with integrable structure, the best known of which are the Camassa-Holm (CH) and Degasperis-Processi (DP) equations and their numerous generalisations. These equations possess stable, non-smooth solutions, called peakons, which, to a large extent, determine the essential properties of solutions, in particular the breakdown of regularity and the onset of shocks (DP). The non-smooth character of these solutions presents a considerable challenge for the question of Lax integrability since the underlying idea of commutativity of partial derivatives no longer holds and one is forced to use ideas of distribution theory mindful that distributional operations require special care for non-linear problems.

In this talk, I will review the pertinent inverse boundary value problems coming from distributional boundary value problems relevant for the peakon sectors of several of these "peakon" equations, with the due emphasis on the approximation theory aspects which play a decisive role in the construction of these solutions. Some of the highlights of this connection involve Stieltljes continued fractions, Hermite-Padé approximations, multipoint Padé approximations, oscillatory kernels of Gantmacher-Krein type and Nikishin systems.

In the second half of my talk I will concentrate on my recent work with Xiangke Chang (Beijing) on the modifed Camassa-Holm equation for which many of the challenges posed by non-smoothness are present.

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Gabor Szegő Lecture 2017

The high oscillation of special functions

Tom Trogdon (University of California, Irvine, USA)

11:30–12:30 Thursday 6th July Sibson LR3

High oscillation in mathematics is ubiquitous and essential. Answering asymptotic questions often relies critically on tools, such as the method of steepest descent, to capture oscillation. In this talk, I will discuss settings where high oscillation governs the fundamental features of a problem but presents intrinsic asymptotic and/or numerical barriers. Examples include short-time behaviour of dispersive PDEs with discontinuous initial data and causal Wiener filters with delay. Special functions, and their related techniques, are often the key to overcoming oscillation-induced difficulties.

Public Lecture

Math is in the eye of the beholder

Andrei Martínez-Finkelshtein (Universidad de Almería, Spain)

18:00–19:00 Tuesday 4th July Sibson LR3

The medical imaging benefits from the advances in many branches of Mathematics, such as constructive approximation, orthogonal polynomials, Fourier and numerical analysis, statistics, to mention a few. At the same time, the needs of the medical diagnostic technology pose new Mathematical challenges. This talk surveys a few problems that have appeared in my collaboration with specialists studying some pathologies of the human eye, in particular, of the cornea.

Contributed Talks

Approximation by operators including the Sheffer-Appell polynomials

Mahvish Ali (Aligarh Muslim University, India)

This talk aims to give a generalisation of Szasz operators including hybrid families of special polynomials. The positive linear operators including the Sheffer-Appell polynomials sequence are constructed. The convergence properties and the order of convergence of these operators are established. Certain explicit examples including mixed type special polynomials are also considered as applications. Graphical representations and error estimates are also established. This talk is first attempt in the direction of finding operators including the hybrid families of special polynomials.

Multiple Laurent orthogonal polynomials on the unit circle and Toda-type systems

Carlos Alvarez (Universidad Pontificia Comillas, Madrid, Spain)

The LU factorisation and the CMV representation have been useful to study the connections between Laurent Orthogonal Polynomials on the real circle and integrable systems of Toda-type. Here we extend the techniques to the multiple case. Recurrence relations, Christoffel-Darboux formulas and connections with Toda are obtained.

On the modified parameters of orthogonal polynomials

Swaminathan Anbhu (Indian Institute of Technology Roorkee, India)

In this work, we consider the three term recurrence relation of certain orthogonal polynomials. By changing the parameters, we obtain the modified three term recurrence relations which are related to the co-dilated and co-recursive polynomials. We investigate the weight function of these modified polynomials and the behaviour of their zeros. The structural relations of the modified polynomials are analysed and the effect of zeros on the Gaussian quadrature formula are discussed.

Jacobi sequences of powers of random variables

Abdessatar Barhoumi (University of Carthage, Tunisia)

We express the Jacobi sequences of the powers of a real valued random variable with all moments, not necessarily symmetric, as functions of the corresponding sequences of the random variable itself. For the power 2, in the symmetric case, the result is known and, with our approach, we give a short purely algebraic proof of it. In particular, for the square of the Gamma distribution, i.e. the 4th power of the standard Gaussian, the result confirms the conjecture that Γ^2 belongs to the polynomial class, but its principal Jacobi sequence grows like n^6, not n^4 as expected.

Zeros of linear combinations of Bessel functions of the first kind

Árpád Baricz (Babeş-Bolyai University, Romania and Óbuda University, Hungary)

In the study of the local zero behaviour of orthogonal polynomials around an algebraic singularity the asymptotic behaviour of neighbouring zeros around the singularity can be described with the zeros of the linear combination of Bessel functions of the first kind. By using Sturm-Liouville theory, we study the behaviour of this linear combination of Bessel functions, thus providing estimates for the zeros in question. The talk is based on the paper arXiv:1610.07309.

Bergman orthogonal polynomials and the Grunsky matrix

Bernhard Beckermann (University of Lille, France)

By exploiting a link between Bergman orthogonal polynomials and the Grunsky matrix probably first observed by Kühnau in 1985, we improve some recent results on strong asymptotics of Bergman polynomials outside the domain G of orthogonality, and on entries of the Bergman shift. We also recall from the literature some links between regularity of the boundary of G, the Grunsky operator and the related conformal maps. Joint work with Nikos Stylianopoulos.

Orthogonal rational functions with poles on the unit circle and at the origin

Kiran Kumar Behera (Indian Institute of Technology Roorkee, India)

In this work, we characterise orthogonal rational functions with poles lying on the unit circle and an additional pole at origin. We derive the recurrence relations satisfied by these rational functions, leading to a generalised eigenvalue problem. The associated measure and quadrature formulas on the unit circle are also discussed.

Some inequalities of oscillating integrals

Jamal Benbourenane (Abu Dhabi University, United Arab Emirates)

In this talk we will consider some inequalities of oscillating integrals. These integrals are available in many applications and their approximations is always sought. The unpredictability of their integrand fluctuations make it harder to predict their sign and predict their behaviour. A concise mathematical approach will be given to explain their global behaviour over a closed interval.

Indeterminate moment problems

Christian Berg (University of Copenhagen, Denmark)

A Hamburger moment problem is characterised either by the moment sequence (s_n) or by two real sequences $(a_n),(b_n)$ with $b_n > 0$. These sequences determine the orthonormal polynomials $P_n, n \geq 0$ via the three-term recurrence relation

$$xP_n(x) = b_n P_{n+1}(x) + a_n P_n(x) + b_{n-1} P_{n-1}(x), n \geq 0$$

with the initial conditions $P_{-1}(x) = 0, P_0(x) = 1$. The indeterminate case is characterised by the convergence of the series $\sum |P_n(z)|^2$ for all complex z, and it leads to a reproducing kernel Hilbert space $\mathscr{E}$ of entire functions with the reproducing kernel

$$K(z,w) = \sum_{n=0}^{\infty} P_n(z)P_n(w) = \sum_{k,l=0}^{\infty} a_{k,l} z^k w^l, \quad z,w \in \mathbb{C}.$$

In the indeterminate case the complete description of all solutions to the moment problem depends on four entire functions A, B, C, D of common order and type called the order and type of the moment problem. The function D is given as

$$D(z) = zK(z,0) = z \sum_{n=0}^{\infty} P_n(z)P_n(0), \quad z \in \mathbb{C},$$

but in general it is difficult to calculate D. It is therefore of some interest to be able to calculate the order and type of the moment problem directly from the coefficients $(a_n),(b_n)$ without calculating first (P_n) and D. This has been achieved in [1, 2] for certain classes of sequences $(a_n),(b_n)$. In the indeterminate case we can consider the two infinite matrices $\mathscr{A} = (a_{k,l}), \mathscr{H} = (s_{k+l})$. We shall see that in certain indeterminate cases but not all the following infinite matrix equations hold: $\mathscr{A}\mathscr{H} = \mathscr{H}\mathscr{A} = I$. I shall report on some of these results and work in progress with Ryszard Szwarc.

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Generating functions for the Bannai-Ito polynomials

Geoffroy Bergeron (University of Montreal, Canada)

A generating function for the Bannai-Ito orthogonal polynomials is derived using the fact that these polynomials are known to be essentially the Racah or $6j$ coefficients of the $\mathfrak{osp}(1|2)$ Lie superalgebra. After the construction of a realisation of this Racah problem in terms of three Dunkl oscillators, the generating function will be obtained using an asymptotic expansion of the associated Racah decomposition.

Szegő transformation and zeros of analytic perturbations of Chebyshev weights

Elías Berriochoa (Universidad de Vigo, Spain)

The study of zeros of orthogonal polynomials with respect to analytic weights on the unit circle has focused the attention of researchers after the classical paper by Nevai and Totik [Orthogonal polynomials and their zeros, *Acta Sci. Math. (Szeged)* **53** (1989), 99–104]. Special mention deserves the contributions of E.B. Saff and B. Simon among others. In this contribution we deal with analytic properties of those orthogonal polynomials which lead to the zero distribution of para-orthogonal polynomials associated with such a kind of measures. Taking into account the Szegő transformation, we study an analogous situation for measures supported on a bounded interval and, as a consequence, results concerning polynomials orthogonal with respect to analytical modifications of the Chebyshev measure of the first kind are deduced.
Our main result establishes a method to obtain an approximation of the nearest zeros to ± 1 of the orthogonal polynomials related to the analytical modifications of the Chebyshev measure of the first kind, that is, $\mathrm{d}\mu(x) = \dfrac{w(x)}{\sqrt{1-x^2}}$. The approximation obtained with a low computational cost has order $\mathscr{O}(n^{-6})$. This method is extended to the first kth zeros closest to ± 1. Furthermore, we discuss the case of analytic modifications of the other three Chebyshev measures and, finally, a complete set of examples is given.

Zeros of Krall polynomials

Oksana Bihun (University of Colorado, Colorado Springs, U.S.A.)

We identify a class of remarkable algebraic relations satisfied by the zeros of the Krall orthogonal polynomials that are eigenfunctions of linear differential operators of order higher than two. Given an orthogonal polynomial family $p_n(x)$, we relate the zeros of the polynomial p_N with the zeros of p_m for each $m \leq N$ (the case $m = N$ corresponding to the relations that involve the zeros of p_N only). These identities are obtained by exacting the similarity transformation that relates the spectral and the (interpolatory) pseudospectral matrix representations of linear differential operators, while using the zeros of the polynomial p_N as the interpolation nodes. The proposed framework generalises known properties of classical orthogonal polynomials to the case of nonclassical polynomial families of Krall type. We illustrate the general result by proving new remarkable identities satisfied by the Krall-Legendre, the Krall-Laguerre and the Krall-Jacobi orthogonal polynomials.

Zeros of exceptional Laguerre polynomials

Niels Bonneux (KU Leuven, Belgium)

I will discuss the asymptotic behaviour of the regular zeros and the exceptional zeros of exceptional Laguerre polynomials. The polynomials are constructed by associating them with two partitions. After contracting by a factor n, the regular zeros are attracted by the zeros of the Bessel function and they are distributed according to the Marchenko-Pastur distribution. The exceptional zeros tend to the zeros of the generalised Laguerre polynomial, provided that the zeros of the generalised Laguerre polynomial are simple. The condition of simple zeros may not be too restrictive because we conjecture that these zeros are indeed simple.

Exploring the zeros of real self-reciprocal polynomials by the Chebyshev polynomials

Vanessa Botta (Universidade Estadual Paulista, Brazil)

In this talk we present four classes of real self-reciprocal polynomials with at most two zeros located outside the unit circle. The behaviour of the zeros of real self-reciprocal polynomials is related with the quasi-orthogonality of a linear combination of Chebyshev polynomials.

Multivariate generalisations of the Chebyshev polynomials of the second kind

Adam Brus (Czech Technical University in Prague, Czech)

The Chebyshev polynomials of the second kind are generalised with use of multivariate symmetric generalisations of trigonometric functions. For such orthogonal polynomials of several variables are shown the recurrence relations by use of generalised trigonometric identities. For dimension three the exact form of recurrence relations is obtained and then used to calculate first ten polynomials. Further the possibility of generalisation which uses antisymmetric multivariate sine function and generalisation of the Chebyshev polynomials of the fourth kind is discussed.

Positivity, extension and integral representations for special functions

Jorge Buescu (Universidade de Lisboa, Portugal)

A holomorphic positive definite function f defined on a horizontal strip of the complex plane are characterised as the Fourier-Laplace transform of a unique exponentially finite measure on $\mathbb{R}$. The classical theorems of Bochner on positive definite functions and of Widder on exponentially convex functions become special cases of this characterisation: they are respectively the real and pure imaginary sections of the complex integral representation. In terms of the corresponding probability measure these are, respectively, the characteristic and moment-generating function. We apply this integral representation to special cases, including the Γ and the ζ functions, and construct explicitly the corresponding measure, thus providing new insight into the nature of complex positive and co-positive definite functions.

Homogeneous q-partial difference equations and some applications

Jian Cao (Hangzhou Normal University, China)

In this talk, we show how to prove identities and evaluate integrals by expanding functions in terms of products of the q-hypergeometric polynomials by homogeneous q-partial difference equations, we also generalise some results of Liu (2015) and Cao (2016). In addition, we generalise multilinear and multiple generating functions for the q-hypergeometric polynomials as applications. Moreover, we deduce some recurring formulas for Ramanujan's integrals, Askey-Roy integrals, Andrews-Askey integrals and moment integrals by the method of homogeneous q-partial difference equations. Finally, we build the relation of Ismail-Zhang type generating functions for the q- hypergeometric polynomials by the method of Ismail and Zhang (2016).

Matrix orthogonal polynomials and time-band limiting

Mirta M. Castro Smirnova (University of Seville, Spain)

In this talk we will consider examples of matrix valued orthogonal polynomials satisfying differential equations (i.e a bispectral situation) in connection with time and band limiting. For a given family of matrix orthogonal polynomials one considers the global operator defined by a full symmetric matrix or an integral operator, given by the truncated inner products. We search for a local operator given by a narrow band matrix or a differential operator (respectively), with simple spectrum, commuting with this operator. The existence of a commuting local operator is very useful to compute numerically the eigenfunctions of the given global operator.

Our study is motivated by the work of Claude Shannon and a series of papers by D. Slepian, H. Landau and H. Pollak at Bell Labs in the 1960's.

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Generalisations of the semi- and fully discrete Lotka-Volterra lattice

Xiaomin Chen (Max Planck Institute for Dynamics and Self-Organization, Göttingen, Germany)

It is well known that the integrable semi-discrete and fully discrete Lotka-Volterra equations possess special Lax pairs involving symmetric orthogonal polynomials. We construct a nonisospectral semi-discrete Lotka-Volterra equation and propose a more general solution of the fully discrete Lotka-Volterra equation. The key point is introducing a more general evolution relation for the moments of symmetric orthogonal polynomials, which involves a "convolution term". Furthermore, we generate corresponding exact ("molecule") solutions, expressed in terms of Hankel-type determinants. Our approach makes use of Hirota bilinear method and determinant techniques.

Determinacy and indeterminacy criteria for moment problems associated with OPS given by a three term recurrence relation

Theodore Chihara (Purdue University Northwest, Hammond campus, USA)

For an OPS satisfying the three term recurrence relation

$$P_n(x) = (x - c_n)P_{n-1}(x) - \lambda_N P_{n-2}(x),$$

$n \geq 1$, $P_0(x) = 1$, $P_{-1}(x) = 0$, $c_n \in \mathbb{R}$, $\lambda_{n+1} > 0$, we find new criteria for determinacy and indeterminacy of the associated Hamburger moment problems under the hypotheses that the coefficients satisfy certain convergence properties, namely, $c_n \to \infty$ and the sequence $\{\lambda_{n+1}/(c_n c_{n+1})\}$ converges to a finite limit, L. Naturally, we are most interested in the case, $L = \frac{1}{4}$ (i.e., the one-quarter class).

Asymptotics of Chebyshev polynomials

Jacob S. Christiansen (Lund University, Sweden)

Given an infinite compact set $\mathsf{E} \subset \mathbb{R}$, the nth Chebyshev polynomial, $T_n(z)$, is the unique monic polynomial of degree n that minimises the sup-norm $\|T_n\|_{\mathsf{E}} =: t_n$ on E. While the lower bound $t_n \geq C(\mathsf{E})^n$ is classical, I shall briefly discuss upper bounds of the form

$$t_n \leq K \cdot C(\mathsf{E})^n$$

for some $K > 0$. Here, $C(\mathsf{E})$ is the logarithmic capacity of E. The main focus of the talk will be on asymptotics of $T_n(z)$ (and t_n). I'll explain how to solve a 45+ year old conjecture of Widom for finite gap subsets of $\mathbb{R}$ and then discuss how one can go far beyond this simple class of subsets. Joint work with Barry Simon (Caltech, Pasadena, USA), Peter Yuditskii (JKU, Linz, Austria) and Maxim Zinchenko (UNM, Albuquerque, USA).

The heat equation for Jacobi matrices

Óscar Ciaurri (Universidad de La Rioja, Spain)

Let J be a Jacobi matrix having a bounded spectral measure. In this talk we will present a solution for the heat equation associated to the Jacobi matrix

$$\frac{\partial u(n,t)}{\partial t} = -Ju(n,t), \qquad u(n,t) = f(n),$$

for an appropriate sequence $f(n)$. It will be given by a family of sequences $W_t f(n)$, with $t \geq 0$, defined by an integral kernel. Moreover, we will check that $W_t f$ generates a semigroup. By using this fact, we will define the negative powers of the Jacobi matrix related to the Jacobi polynomials and a Hardy-Littlewood inequality for them will be proved.

Binomial and logarithmic Gegenbauer expansions for the even-dimensional polyharmonic equation

Howard Cohl (NIST, Gaithersburg, USA)

On even-dimensional Euclidean space for integer powers of the Laplacian greater than or equal to the dimension divided by two, a fundamental solution of the polyharmonic equation has binomial and logarithmic behaviour. Gegenbauer polynomial expansions of these fundamental solutions are obtained through a limit process applied to Gegenbauer expansions of a power-law fundamental solution of the polyharmonic equation. For the Gegenbauer logarithmic fundamental solution expansion, we use parameter derivatives applied to the Gegenbauer expansion of a power-law fundamental solution for the polyharmonic equation. By combining these results with previously derived azimuthal Fourier series expansions for these binomial and logarithmic fundamental solutions, we obtain addition theorems for the azimuthal Fourier coefficients in Vilenkin polyspherical standard geodesic polar coordinates and generalised Hopf coordinates.

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q-Polynomials for non-standard parameters. Orthogonality and new identities.

Roberto S. Costas-Santos (Universidad de Alcalá, Spain)

We present the orthogonality os some families of q-polynomials for non-classical parameters. We also will present some new identities related with these families. We will consider the Al-Salam-Carlitz, big q-Jacobi and q-Laguerre polynomials.

Asymptotics of the index distribution for the Gaussian unitary ensemble

Dan Dai (City University of Hong Kong, China)

We study the index n_+ of an $n \times n$ matrix belonging to the Gaussian unitary ensemble, which is the number of its positive eigenvalues. Based on a matrix model with a jump at the origin, we derive the asymptotics of the index distribution as the matrix size n becomes large, with $n_+ = cn$ and $0 \le c \le 1$. This is a work in progress with Shuai-Xia Xu and Yu-Qiu Zhao.

Asymptotic expansions for multiple Gamma functions

Sourav Das (Indian Institute of Technology Roorkee, India)

In this work asymptotic expansions for multiple Gamma functions are discussed. Consequences of two variations of these asymptotic expansions are outlined leading to approximation of corresponding multiple Gamma functions by specific type of special functions. Applications for such approximations are outlined.

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Asymptotics for Hankel determinants with discontinuous Hermite weights

Alfredo Deaño (University of Kent, UK)

We consider the problem of finding large n asymptotic expansions for $n \times n$ Hankel determinants which are constructed from moments of discontinuous Hermite weights on the real line. Such Hankel determinants arise in the study of special function solutions of the Painlevé IV differential equation and in the thinning process of GUE ensembles, and they can be studied using the Deift-Zhou method of steepest descent for Riemann-Hilbert problems.

Sobolev extremal polynomials

Abel Díaz González (Carlos III University of Madrid, Spain)

Extremal polynomials with respect to a Sobolev-type p-norm, with $1 < p < \infty$ and measures supported on compact subsets of the real line, are considered. For a wide class of such extremal polynomials with respect to measures mutually singular (i.e. supported on disjoint subsets of the real line), it is proved that their critical points are simple and contained in the interior of the convex hull of the support of the measures involved, the asymptotic critical point distribution is studied. We also find the nth root asymptotic behaviour of the corresponding sequence of Sobolev extremal polynomials derivatives.

Factorisation of stochastic Jacobi matrices into two stochastic factors and Darboux transformations

Manuel Dominguez de la Iglesia (Universidad Nacional Autonoma de Mexico, Mexico)

We consider UL (and LU) decompositions of the one-step transition probability matrix of a random walk on the nonnegative integers with the condition that both upper and lower triangular matrices from the factorisation are also stochastic matrices. We give conditions on the free parameter of the UL factorisation in terms of certain continued fraction such that this stochastic factorisation is always possible. We finally give some examples. Joint work with F. A. Grünbaum

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A coupling problem for entire functions

Jonathan Eckhardt (University of Vienna, Austria)

Solutions of completely integrable systems can typically be recovered by means of solving Riemann-Hilbert problems. When the underlying spectrum is purely discrete though, the role of these Riemann-Hilbert problems is taken by coupling problems for entire functions. I will discuss such a coupling problem that arises in connection with the Camassa-Holm equation.

Chebyshev problems on a circular arc

Benjamin Eichinger (Johannes Kepler University, Linz, Austria)

We consider the Chebyshev polynomials, T_n, on an arc, A_α, of the unit circle, i.e., the monic polynomials of degree at most n minimising the sup-norm, $\|T_n\|_{A_\alpha}$. Thiran and Detaille found an explicit formula for the asymptotics of $\|T_n\|_{A_\alpha}$, which disproved a conjecture of Widom. We give the Szegő-Widom asymptotics of the domain explicitly. That is, the limit of the properly normalised extremal functions T_n. Moreover, we solve a similar problem with respect to the upper envelope of a family of polynomials uniformly bounded on A_α. Our computations show that in the proper normalization the limit of the upper envelope is the diagonal of a reproducing kernel of a certain Hilbert space of analytic functions. Similarly, we study the *polynomial Ahlfors problem*, i.e., we maximise $|P'(z)|$ at a fixed point $z \in \mathbb{C} \setminus A_\alpha$ in the class of polynomials of degree at most n that are uniformly bounded on A_α and vanish at the given point. The corresponding asymptotics can again be given in terms of an explicit reproducing kernel.

The Swallowtail integral in the highly oscillatory region

Chelo Ferreira (Universidad de Zaragoza, Spain)

We consider the swallowtail integral

$$\Psi(x,y,z) := \int_{-\infty}^{\infty} \exp\left\{ \mathrm{i}(t^5 + xt^3 + yt^2 + zt) \right\} \mathrm{d}t,$$

for large values of one variable and bounded values of the other two variables. The integrand of the swallowtail integral oscillates wildly in this region and the asymptotic analysis is subtle. The standard saddle point method is complicated and then we use the simplified saddle point method introduced in [López el al., 2009]. The analysis is more straightforward with this method and it is possible to derive complete asymptotic expansions of $\Psi(x,y,z)$ when one variable is large. The asymptotic analysis requires the study of different regions for the argument of the asymptotic variable that are separated by the Stokes lines. The asymptotic approximation is given in terms of elementary functions of x, y and z. The accuracy of the approximations is illustrated with some numerical experiments.

An algebraic interpretation of the q-Meixner polynomials

Julien Gaboriaud (Université de Montréal, Canada)

An algebraic interpretation of the q-Meixner polynomials is presented. These polynomials are seen to appear as matrix elements of unitary q-pseudorotations representations on q-oscillator states. These unitary operators are constructed by q-exponentiation of the generators of the $\mathscr{U}_q(\mathfrak{su}(1,1))$ algebra. The interpretation as matrix elements allows for an elegant derivation of the properties of these polynomials: orthogonality relations are obtained, structure relations (backward and forward relations) are obtained, naturally leading to a difference relation, dual backward and forward relations are obtained, naturally leading to a recurrence relation. A duality property is observed, allowing the derivation of a new set of these type of structure relations. Generating functions of two different types are finally obtained. The framework constructed in the hereabove work paves the way to an algebraic interpretation of the multivariate q-Meixner polynomials.

Wronskians with classical polynomial entries

María Ángeles García-Ferrero (ICMAT, Madrid, Spain)

Karlin and Szegő studied Wronskian determinants whose entries are orthogonal polynomials, working out the number of real zeros for a Wronskian of consecutive polynomials. Krein and Adler independently gave necessary and sufficient conditions characterising the sequences such that the Wronskian has constant sign, which turns out to be essential to characterise the regularity of the transformed potential after a sequence of Darboux transformations. Expanding Adler's oscillatory approach, we are able to prove an elegant formula that counts the number of real zeros of the Wronskian of an arbitrary sequence of eigenfunctions, thus generalising all above results. As a particular case, the formula applies to any Wronskian whose entries are classical orthogonal polynomials. Applications of this formula include, for instance, determination of the number of real zeros and poles of the rational solutions to Painlevé equations. This is a joint work with David Gómez-Ullate [1].

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Efficient computation of classical orthogonal polynomials for large orders

Amparo Gil (Universidad de Cantabria, Spain)

Efficient computational schemes for evaluating Hermite, Laguerre and Jacobi polynomials for large orders are discussed. The computation is based on the use of different asymptotic approximations (in terms of elementary, Airy or Bessel functions), recurrence relations and local Taylor series, depending on the parameter region. The resulting algorithms extend the range of validity of previous approaches and fill a gap in the numerical software libraries for mathematical functions. This is a joint work in collaboration with Javier Segura and Nico M. Temme.

Smallest singular value distribution and large gap asymptotics for products of random matrices

Manuela Girotti (Colorado State University, Fort Collins, USA)

We study the distribution of the smallest singular eigenvalues for the finite product of certain random matrix ensemble, in the limit where the size of the matrices becomes large. The limiting distributions that we will study can be expressed as Fredholm determinants of certain integral operators, and generalise in a natural way the extensively studied hard edge Bessel kernel determinant. We will express such quantities in terms of a 2×2 Riemann-Hilbert problem, and use this representation to obtain so-called large gap asymptotics.

From Schoenberg coefficients to Schoenberg matrix functions

Jean Carlo Guella (Universidade de São Paulo-ICMC, Brazil)

A function K mapping a product space $Z \times Z$ into $M_l(\mathbb{C})$ is termed a *positive definite kernel* on Z of order l if

$$\sum_{\mu,\nu=1}^{n} c_\mu K(z_\mu, z_\nu)\overline{c_\nu}^t \geq 0,$$

for $n \geq 1$, distinct points $z_1, z_2, \ldots, z_n$ in Z and $c_1, c_2, \ldots, c_n \in \mathbb{C}^l$. The strict positive definiteness of a positive definite kernel K demands that the inequalities above be strict when at least one of the c_i is non null. In 1942, I.J. Schoenberg published his seminal paper characterising the continuous and isotropic positive definite kernels on spheres of order 1: $f : [-1,1] \to \mathbb{C}$ is continuous and the kernel $(x,y) \in S^d \times S^d \to f(x \cdot y)$ is positive definite, where $\cdot$ is the usual inner product in $\mathbb{R}^{d+1}$ if, and only if,

$$f(t) = \sum_{k=0}^{\infty} a_k^d P_k^d(t), \quad t \in [-1,1],$$

where all the coefficients a_k^d are nonnegative, P_k^d denotes the usual Gegenbauer polynomial of degree k attached to the rational number $(d-1)/2$ and $\sum_k a_k^d P_k^d(1) < \infty$. The characterisation for strict positive definiteness in the case $d \geq 2$ appeared in 2003 in a work of Menegatto et all: a positive definite kernel from Schoenberg's class is strictly positive definite if, and only if, the index set $\{k : a_k^d > 0\}$ contains infinitely many even and infinitely many odd integers. The aim of this talk is to give a generalisation of this two results for kernels $((u,x),(v,y)) \in (X \times S^d)^2 \to f(u,v,x \cdot y) \in M_l(\mathbb{C})$, where X is a non empty set and each coordinate of each matrix function $f(u,v,\cdot)$ is continuous in $[-1,1]$.

On a generalisation of Jacobi's Elegantissima.

Luc Haine (Université Catholique de Louvain, Belgium)

The famous Belgium cosmologist Georges Lematre concludes his lessons on the pendulum dating from 1956-1957 by "The theory that we have exposed can be considered as a generalisation of Jacobi's Elegentissima". We will try to elucidate the mystery, which is related to Poncelet theorem.

Connection formulas for the Ablowitz-Segur solutions of the inhomogeneous Painlevé II equation

Weiying Hu (City University of Hong Kong, China)

We consider the second Painlevé II equation

$$u''(x) = 2u^3(x) + xu(x) - \alpha,$$

where α is a nonzero constant. Using the Deift-Zhou nonlinear steepest descent method for Riemann-Hilbert problems, we rigorously prove the asymptotics as $x \to \pm\infty$ for both the real and purely imaginary Ablowitz-Segur solutions, as well as the corresponding connection formulas. We also show that the real Ablowitz-Segur solutions have no real poles when $\alpha \in (-\frac{1}{2}, \frac{1}{2})$.

Asymptotic behaviour and electrostatic properties of zeros of Freud-Sobolev type orthogonal polynomials

Edmundo J. Huertas (Universidad Politécnica de Madrid, Spain)

In this contribution we consider sequences of monic polynomials orthogonal with respect to the non-standard Freud-like inner product involving a quartic potential

$$\langle p,q \rangle = \int_{\mathbb{R}} p(x)q(x)\exp(x^4)\,\mathrm{d}x + M_0 p(0)q(0) + M_1 p(0)q(0).$$

We analyse some properties of these polynomials, such as the ladder operators and the holonomic equation that they satisfy and, as an application, we give an electrostatic interpretation of their zero distribution in terms of a logarithmic potential interaction under the action of an external field. The asymptotic behaviour and interlacing properties of their zeros in terms of their dependence on M_0 and/or M_1 are given.

Matrix-valued Chebyshev polynomials of several variables

Daan Huybrechs (KU Leuven, Belgium)

Chebyshev polynomials of the first and second kinds are well-known univariate orthogonal polynomials. They are closely linked to Fourier series, and in that context each kind of Chebyshev polynomial is further linked with a distinct symmetry: the first kind polynomials correspond to even symmetry (cosine series), and the second kind to odd symmetry (sine series). In this talk we consider more general symmetry groups, associated with lattices in multiple dimensions. Using representation theory, we show that Fourier analysis on these lattices naturally decomposes into symmetric parts, each of which corresponds to a different kind of Chebyshev polynomial. Some of these orthogonal polynomials are both multivariate and matrix-valued.

Properties of orthogonal polynomials with respect to semi-classical perturbations of classical weights

Kerstin Jordaan (University of South Africa, Pretoria, South Africa)

In this talk I will discuss properties of orthogonal polynomials with respect to semi-classical perturbations of classical weights. I will show that for some semi-classical weights, in particular the semi-classical Laguerre and generalised Freud weight, the coefficients in the three term recurrence relation satisfied by the polynomials associated with the weight can be expressed in terms of Wronskians of parabolic cylinder functions which arise in the description of special function solutions of Painlevé equations. Unique, positive solutions of the nonlinear difference equation satisfied by the recurrence coefficients exist for all real values of the parameter involved in the semi-classical perturbation but these solutions are very sensitive to the initial conditions. I will derive a structural relation and second-order linear ordinary differential equation satisfied by generalised Freud polynomials and analyse the asymptotic behaviour of the polynomials and recurrence coefficients in two different contexts. Properties and bounds for the zeros of semi-classical Laguerre and generalised Freud polynomials will also be discussed.

Discrete Fourier calculus of Weyl orbit functions

Michal Juránek (Czech Technical University in Prague, Czech)

Various types of special functions of compact simple Lie groups, or equivalently, of their underlying root systems, form a branch of study in modern Lie theory. The ten types of orbit functions with their corresponding affine Weyl groups are considered. Our intention is to formulate an approach that unifies these ten types of orbit functions. Finite subsets of Weight lattices and dual Weight lattices are formulated to serve as a set of labels and a sampling grid, respectively. The complete sets of discretely orthogonal orbit functions over the sampling grids are found and the corresponding discrete Fourier transforms are formulated. The standard one-dimensional discrete cosine, sine and Fourier transforms form special cases of the presented transforms.

Inequalities for series in ratios of Gamma and q-Gamma functions

Sergei Kalmykov (Shanghai Jiao Tong University, Shanghai, China)

Mainly we will discuss power series with general nonnegative coefficients dependent on an additional parameter included as an argument of shifted factorial or Gamma function and their q-analogues. Several types of such series will be considered. We will demonstrate how non-negativity or logarithmic concavity of the coefficients leads to q-log-convexity or q-log-concavity as well as multiplicative convexity or concavity for the sum of the series. Applications to modified q-Bessel functions and other q-hypergeometric functions will be given as well. This is based on a joint work with Dmitrii Karp.

A characterisation of Askey-Wilson polynomials

Maurice Kenfack Nangho (University of Pretoria, South Africa)

We characterise polynomials that satisfy a divided-difference equation of the form

$$\pi(x(s))\mathbb{D}_x^2 P_n(x(s)) = \sum_{j=-2}^{2} a_{n,n+j} P_{n+j}(x(s)), \quad a_{n,n-2} \neq 0, \quad n = 2,3,\ldots \tag{1}$$

where $x(s) = c_1 q^{-s} + c_2 q^s + c_3$ or $x(s) = c_4 s^2 + c_5 s + c_6$, $\pi(x)$ is a polynomial of degree at most 4 and $\mathbb{D}_x$ is the divided-difference operator

$$\mathbb{D}_x f(x(s)) = \frac{f(x(s+\frac{1}{2})) - f(x(s-\frac{1}{2}))}{x(s+\frac{1}{2}) - x(s-\frac{1}{2})}.$$

We show that the only monic orthogonal polynomials $\{P_n\}_{n=0}^{\infty}$ that satisfy (1) are Askey-Wilson polynomials and their special or limiting cases. This completes, proves and extends a conjecture by Ismail [1, 24.7.9] concerning a structure relation satisfied by Askey-Wilson polynomials.

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Representations of harmonic oscillator Lie algebra and 2-dimensional Hermite polynomials

Subuhi Khan (Aligarh Muslim University, India)

Orthogonal polynomials in one variable are irreplaceable tools for constructing quadrature formulae of different types. A similar situation exists in the case of 2-dimensional orthogonal polynomials. However, it should be noted that while orthogonal polynomials in one variable already have numerous and varied applications in many fields of science, the theory of orthogonal polynomials in two or more variables is applied insufficiently widely. The theory of group representations and its relation to special functions provide a powerful tool to the development of mathematical physics. Special functions appear as basis vectors and matrix elements corresponding to local multiplier representations of Lie groups. In this work, certain results for a recently introduced one parameter generalisation of the 2-dimensional Hermite polynomials are established by using the representation theory of the Harmonic oscillator Lie group $G(0;1)$. These results are derived by making use of the irreducible representations of the corresponding Lie algebra. Certain examples involving other related orthogonal polynomials are also considered.

Multivariable matrix-valued orthogonal polynomials

Erik Koelink (Radboud Universiteit, Nijmegen, The Netherlands)

The intimate relationship between group theory enables us to introduce multivariable matrix-valued orthogonal polynomials by studying matrix-valued spherical functions for higher rank symmetric spaces. The details are worked out for the case of $(\mathrm{SU}(n+1) \times \mathrm{SU}(n+1), \mathrm{SU}(n+1))$ for which we derive several explicit properties, such as being joint eigenfunctions of an algebra of matrix-valued differential operators. The case $n = 1$ gives back single-variable matrix-valued orthogonal polynomials, whereas the case $n = 2$ gives matrix-valued analogues of the orthogonal polynomials on the interior of Steiner's hypocycloid introduced in 1974 by Koornwinder. This is joint work with Maarten van Pruijssen (University of Paderborn, Germany) and Pablo Román (University of Nacional Cordóba, Argentina).

Some steps to a q-Askey scheme of double affine Hecke algebras

Tom Koornwinder (University of Amsterdam, The Netherlands)

Cherednik's double affine Hecke algebras (DAHA's) provide the algebraic setting for the non-symmetric variants of special functions associated with root systems. The Askey-Wilson DAHA is well studied. Its spherical subalgebra is closely connected with Zhedanov's Askey-Wilson algebra. This latter algebra is at the top of a q-Askey scheme of such algebras. A similar scheme of DAHA's is far from obvious. The talk will discuss some arrows in such a scheme. One aspect is the breaking of symmetry in the parameters when passing from the symmetric to the non-symmetric Askey-Wilson case, which is reflected when taking limits of DAHA's. The talk builds on an earlier paper jointly with Bouzeffour and on recent, yet unpublished work with Marta Mazzocco.

Relative strong Szegő theorem

Rostyslav Kozhan (Uppsala University, The Netherlands)

We establish a relative version of the Strong Szegő Limit Theorem for Toeplitz determinants. Our approach is based on the theory of orthogonal polynomials on the unit circle. As an application we obtain the Central Limit Theorem for linear statistics of the eigenvalues of orthogonal polynomial ensembles of random unitary matrices. Our method can handle orthogonality measures with essential support on the full circle or a single arc that satisfy the Lopez conditions. In particular, this allows the measure to have a singular component within or outside of the arc. Joint work with M.Duits.

Multi-point Taylor expansions in the approximation of integrals and solutions of linear differential equations

Jose Lopez (State University of Navarra, Spain)

Two-point Taylor expansions of analytic functions were discussed in [Lopez and Temme, 2002], and later generalised to multi-point Taylor expansions in [Lopez and Temme, 2004]. Since then, we have investigated applications of this theory in two different fields: (i) integrals and (ii) linear differential equations. We have investigated the use of multi-point Taylor expansions as an alternative to the use of the standard Taylor expansion in the approximation of integrals and in the approximation of solutions of linear differential equations. In the case of integrals, the key point is the approximation of part of the integrand by a convergent or asymptotic expansion in a certain subset of the integration interval D. In the case of differential equations, the key point is the representation of the solution as a convergent series expansion in the interval of definition D of the equation. In both cases, a multi-point Taylor expansion is more efficient than the standard Taylor expansion, as the domain of convergence (a Cassini's disk) is better adapted to the above mentioned interval D than the standard Taylor series. We give several examples of approximation of integrals and solutions of differential equations, and applications to the approximation of special functions.

Riemann-Hilbert problems for matrix orthogonal polynomials and discrete matrix Painlevé equations

Manuel Mañas (Universidad Complutense de Madrid, Spain)

In this talk we consider a Riemann-Hilbert problem description for matrix orthogonal polynomials in the real line and in the unit circle. Given matrix of measures characterised by a Pearson equation we find for the recursion coefficients non Abelian matrix versions of the discrete PI and PII. (Joint work with A Branquinho and AF Moreno)

Structure relations for moments and orthogonal polynomials in two variables associated with classical moment functionals

Misael Marriaga (Universidad Carlos III de Madrid, Spain)

The bivariate classical moment functional u satisfies a system of two distributional partial differential equations with polynomial coefficients a, b and c of degree at most 2, and d, e of degree equal to 1. In this work, we show that the moments of u satisfy matrix three term relations, and give an explicit expression for the matrix coefficients in terms of the polynomials a, b, c, d and e. We also use these matrices to compute the parameters in other structure relations satisfied by the orthogonal polynomials associated with u.

Around operators not increasing the degree of polynomials

Teresa A. Mesquita (Universidade do Porto, Portugal)

We present a generic operator J simply defined as a linear map not increasing the degree from the vectorial space of polynomial functions into itself and we address the problem of finding the polynomial sequences that coincide with the (normalized) J-image of themselves. The technique developed assembles different types of operators. We will also provide examples where the results are applied to the case where J's expansion is limited to three terms. This is a joint work with Pascal Maroni.

Bispectrality and exceptional orthogonal polynomials

Robert Milson (Dalhousie University, Halifax, Canada)

Exceptional Hermite polynomials are complete families of orthogonal polynomials that arise as eigenfunctions of a Sturm-Liouville problem. However, unlike their classical counterparts, the degree sequence of an exceptional family has a finite number of exceptional degrees; i.e., there are gaps in the degree sequence. It is now known that all exceptional operators can be obtained by "dressing" their classical counterparts. By introducing a bispectral anti-homomorphism it is possible to show that these commutation transformations also have a well-defined action on the 3-term recurrence relations of the classical OP families and give rise to higher-order recurrence relations for the exceptional families. We illustrate these ideas for the class of exceptional Hermite polynomials.

Fourier series of Gegenbauer-Sobolev polynomials

Judit Mínguez (University of La Rioja, Spain)

We study the partial sum operator for a Sobolev-type inner product related to the classical Gegenbauer polynomials. A complete characterisation of the partial sum operator in a appropriate Sobolev space is given. Moreover, we analyse the convergence of the partial sum operators.

On symmetric $(1,1)$-coherent pairs and Sobolev orthogonal polynomials for Hermite $(1,1)$-coherent pairs

Luis Alejandro Molano Molano (Universidad Pedagógica y Tecnológica de Colombia, Duitama, Colombia)

In this talk we consider the symmetric positive definite linear functionals u and v, and we assume that there exist sequences of non-zero real numbers $\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$, with $a_n \neq 0$, such that

$$\frac{P'_{n+3}(x)}{n+3} + a_n \frac{P'_{n+1}(x)}{n+1} = R_{n+2}(x) + b_n R_n(x), \quad n \geq 0, \tag{2}$$

where $\{P_n\}_{n\in\mathbb{N}}$ and $\{R_n\}_{n\in\mathbb{N}}$ will denote the corresponding MOPS for u and v, respectively. In this case, (u,v) is said to be a *symmetric* $(1,1)-$*coherent pair*. By using of symmetrisation process, we will describe the classification of the symmetric $(1,1)-$coherent pairs of measures i.e. the corresponding sequences of orthogonal polynomials satisfy (2).

On the other hand, we consider the symmetric $(1,1)-$coherent pair of measures (μ_0,μ_1) with $\mathrm{d}\mu_0 = \exp(-x^2)\,\mathrm{d}x$ and $\mathrm{d}\mu_1 = \dfrac{x^2+a}{x^2+b}\exp(-x^2)\,\mathrm{d}x,\ a,b\in\mathbb{R}^+,\ a\neq b$, and we will consider the monic polynomials $\{S_n^\lambda\}_{n\geq 0}$ orthogonal with respect to Sobolev inner product

$$\langle p,q\rangle_S = \int_{\mathbb{R}} p(x)q(x)\,\mathrm{d}\mu_0 + \lambda\int_{\mathbb{R}} p'(x)q'(x)\,\mathrm{d}\mu_1,\ \ \lambda>0 \tag{3}$$

In this way, we will show the behaviour of the sequences $\{a_n\}$, $\{b_n\}$ and $\{\eta_n(\lambda)\}$ when $n\to\infty$, we will study relative asymptotics for Sobolev scaled polynomial and we will obtain Mehler-Heine type formulas. (*This is a joint work with F. Marcellán and H. Dueñas*)

Special functions and shock soliton solutions of q-viscous Burgers equation

Sengul Nalci Tumer (Izmir Institute of Technology, Turkey)

A new type of heat equation with non-symmetric q-extended diffusion term, which we called the q-diffusive heat equation is introduced. By using the Cole-Hopf transformation we find the corresponding q-viscous Burgers' equation. We study several classes of exact solutions: polynomial and shock soliton type. The polynomial solutions are obtained as generalised Kampe de Feriet polynomials written in terms of Bell polynomials. The generating function for these polynomials is obtained by using dynamical symmetry and the Zassenhaus formula. We get one, two and multiple shock soliton solutions and study their mutual interactions for different values of q. We found that due to specific dependence of the group velocity on wave number, in addition to fusion of the solitons as in usual Burgers equation, a new process of fission of shock solitons with higher amplitude takes place. The "semiclassical" expansion of these equations in terms of Bernoulli polynomials is derived.

Uniform asymptotic smoothing of the higher order Stokes phenomenon

Gergő Nemes (University of Edinburgh, UK)

Exponentially small terms in asymptotic expansions of complex functions can appear suddenly across certain rays. Dingle's non-rigorous final main rule states that half the discontinuity occurs on reaching those rays and half on leaving them the other side. It was demonstrated by Berry in 1989 that these changes are not discontinuous but happen smoothly and according to a universal law. This universal law is also in agreement with Dingle's result. In this talk, we demonstrate, through the example of the classical Stirling asymptotic expansion of the gamma function, that Dingle's rule does not always hold and find an analogue of Berry's smoothing law for these cases.

(Breakdown of) sine kernel universality for Dyson's Brownian motion with deterministic initial conditions

Thorsten Neuschel (Université Catholique de Louvain, Belgium)

It is well known that in many cases the eigenvalues of large random matrices show a universal local behaviour, which typically only depends on few characteristics of the considered matrices. In this talk we deal with random Hermitian matrices of the form

$$Y(t) = M + \sqrt{t}H,$$

where M is a deterministic $n\times n$ Hermitian matrix, $t>0$ and H is an $n\times n$ Hermitian matrix sampled from the Gaussian unitary ensemble (GUE). The matrix $Y(t)$ has the same distribution as a Brownian motion on the space of Hermitian matrices starting at M. The corresponding eigenvalue process, called Dyson's Brownian motion, can also be described by non-intersecting Brownian motions with initial conditions given by the eigenvalues of the matrix M. Let us assume that the empirical eigenvalue distributions $(\mu_n)_n$ of M converge weakly as $n\to\infty$ to some absolutely continuous probability measure μ on $\mathbb{R}$ with compact support. If the density of μ at some point vanishes fast enough, then this vanishing will be visible in the limiting distributions of the eigenvalues of $Y(t)$ for a certain amount of time

$t < t_{cr}$, where we call $t_{cr} > 0$ the critical time. If the density does not vanish or if the vanishing is sufficiently slow, then for the critical time we have $t_{cr} = 0$. Our aim is to study sine kernel universality in the bulk for the local behaviour of the eigenvalues of $Y(t)$ for postcritical times $t > t_{cr}$ and in case that $t_{cr} = 0$ we will explore the situation in which this universal behaviour breaks down for times close to the critical time. The proofs rely on the asymptotic analysis of certain double contour integral representations. The results presented are based on joint work with Tom Claeys and Martin Venker and they are part of a project which is still in progress.

Convolution properties of polynomials associated with Stirling numbers

Bruce O'Neill (Milwaukee School of Engineering)

Stirling numbers of the first kind are used primarily to enumerate configurations of cycle decompositions of permutations. We define sequences of polynomials using rearrangements of Stirling numbers based on counting fixed points of permutations. We study some convolution properties of these sequences, and show there are similar formulae for polynomials constructed from Stirling numbers of the second kind and for some well-known sequences of polynomials.

Solving PDEs on triangles using multivariate orthogonal polynomials

Sheehan Olver (Imperial College, London, UK)

By using a hierarchy of Jacobi-like weights, multivariate orthogonal polynomials on the triangle can give rise to sparse representations of general linear partial differential operators, leading to the efficient solution of partial differential equations such as the Helmholtz and Poisson equations. This generalises the approach of the ultraspherical spectral method, which reduces ordinary differential equations to banded operators. We also discuss general aspects of using multivariate orthogonal polynomials in computations, e.g., a multivariate analogue of Clenshaw's algorithm for evaluating expansions.

Asymptotics for Laguerre-type polynomials and quadrature rules

Peter Opsomer (KU Leuven, Belgium)

Generalised Laguerre-type orthogonal polynomials are orthogonal with respect to the weight function

$$w(x) = x^{\alpha} \mathrm{e}^{-Q(x)}, \quad Q(x) = \sum_{k=0}^{m} q_k x^k, \quad \alpha > -1, \quad q_m > 0,$$

on the interval $[0, \infty)$. Based on a non-linear steepest descent analysis of a Riemann-Hilbert problem, we efficiently compute an arbitrary number of higher order terms in their asymptotic expansions as the degree n increases to ∞ in every region of the complex plane. This is readily extended to Hermite-type weights of the form $\exp(-\sum_{k=0}^{m} q_k x^{2k})$ on $(-\infty, \infty)$, and to general non-polynomial functions $Q(x)$ using contour integrals. Via a Newton method, the expansions may readily be used to compute Gauss-Laguerre quadrature rules in a lower computational complexity than based on the recurrence relation, and with improved accuracy for large degree. We will also describe how to obtain explicit expressions for the asymptotic expansions of the nodes and weights themselves for generalised Laguerre-type quadrature rules, as well as Hermite and Jacobi.

Moment representations of type I X_2 exceptional Laguerre polynomials

John Osborn (Baylor University, USA)

The X_m exceptional orthogonal polynomials (XOP) form a complete set of eigenpolynomials for a differential equation. Despite being complete, the XOP set does not contain polynomials of every degree. Thereby, the XOP escape the Bochner classification theorem. In literature two ways to obtain XOP have been presented. When $m = 1$, Gram–Schmidt orthogonalisation of a so-called “flag” was used. For general m, the Darboux transform was applied. We present a possible flag for the X_m exceptional Laguerre polynomials. There is a large degree of freedom in doing so. Further, we derive determinantal representations of the X_2 exceptional Laguerre polynomials involving certain adjusted moments of the exceptional weights. We find a recursion formula for these adjusted moments. The particular canonical flag we pick keeps both the determinantal representation and the moment recursion manageable.

Generalised Stieltjes functions

Henrik Pedersen (University of Copenhagen, Denmark)

In this talk we deal with the so-called generalised Stieltjes functions of order $\lambda > 0$. These are the functions f representable as

$$f(x) = \int_0^\infty \frac{\mathrm{d}\mu(t)}{(x+t)^\lambda} + c \quad \text{for } x > 0,$$

where μ is a positive measure making the integral convergent and $c \geq 0$. The case of $\lambda = 1$ corresponds to the (ordinary) Stieltjes functions appearing in many areas in analysis. According to a classical result of Widder f is a Stieltjes function if and only if the functions $c_k(f)(x) = (x^k f(x))^{(k)}$, $k = 0, 1, \ldots$ are completely monotonic. Recently Sokal extended this result to generalised Stieltjes functions in terms of a certain doubly indexed sequence of operators. The main purposes of this talk are (1) to relate Sokals operators to complete monotonicity of a single sequence of functions $c_k^\lambda(f)$ and (2) to characterise the first n of these functions being completely monotonic in terms of properties of the representing measure of the completely monotonic function f. The talk is based on joint work with Stamatis Koumandos

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Uniform convergent expansions of some special functions in terms of elementary functions

Ester Pérez-Sinusía (University of Zaragoza, Spain)

Power series expansions and asymptotic expansions of the special functions of mathematical physics have the important property of being given in terms of elementary functions: powers or inverse powers of a certain variable z and, eventually, other elementary functions. But they have the inconvenience that, in general, they are not uniformly valid for all values of z: power series fail for large values of $|z|$, whereas asymptotic expansions fails for small values of $|z|$. In this work we derive convergent expansions of some special functions (Bessel, incomplete gamma and beta functions) in terms of elementary functions that hold uniformly in z in a large region of the complex plane that include small and large values of $|z|$. Error bounds for these expansions are given. The starting point is an integral representation of the considered functions.

Bivariate Racah and q-Racah polynomials; hidden symmetry and alternate bases

Sarah Post (University of Hawaii at Mãnoa, USA)

In this talk we discuss the hidden symmetries associated with the bivariate Racah and q-Racah polynomials. The polynomials arise as expansion coefficients between bases diagonalizing a pair of maximum abelian subalgebras. We will discuss the classes of possible expansions and the connection to a higher-rank generalisation of Leonard pairs.

Hardy Spaces and inclusion properties of generalised fractional integral operator

Jugal Prajapat (Central University of Rajasthan, India)

In this talk, we introduce a new fractional integral operator in unit disk which is a generalisation of the Srivastava-Owa fractional operator. Conditions are obtained for such a generalised fractional integral operator to be bounded in the Hardy space and space of bounded analytic functions.

Basic hypergeometric polynomials having zeros on the unit circle and related orthogonal polynomials.

Alagacone Ranga (Universidade Estadual Paulista, Brazil)

The sequence $\{{}_2\phi_1(q^{-k}, q^{b+1}; q^{-\bar{b}-k+1}; q, q^{-\bar{b}+1/2}z)\}_{k\geq 0}$ of basic hypergeometric polynomials is known to be orthogonal on the unit circle with respect to the weight function $|(q^{1/2}\mathrm{e}^{\mathrm{i}\theta}; q)_\infty/(q^{b+1/2}\mathrm{e}^{\mathrm{i}\theta}; q)_\infty|^2$. This result, where one must take the parameters q and b to be $0 < q < 1$ and $\Re(b) > -\frac{1}{2}$, is due to P.I. Pastro. Here, we deal with the orthogonal polynomials $\hat{\Phi}_n(b;.)$ and $\check{\Phi}_n(b;.)$ on the unit circle with respect to the two parametric families of weight functions $\hat{\omega}(b;\theta) = |(\mathrm{e}^{\mathrm{i}\theta}; q)_\infty/(q^b\mathrm{e}^{\mathrm{i}\theta}; q)_\infty|^2$ and $\check{\omega}(b;\theta) = |(q\mathrm{e}^{\mathrm{i}\theta}; q)_\infty/(q^b\mathrm{e}^{\mathrm{i}\theta}; q)_\infty|^2$, where $0 < q < 1$ and $\Re(b) > 0$. With the use of the basic hypergeometric polynomials ${}_2\phi_1(q^{-k}, q^b; q^{-\bar{b}-k+1}; q, q^{-\bar{b}+1}z)$, $k \geq 0$, which have zeros on the unit circle, simple expressions for the (monic) polynomials $\hat{\Phi}_n(b;.)$ and $\check{\Phi}_n(b;.)$, their norms, the associated Verblunsky coefficients and also the respective Szegő functions are found.

Semi-classical properties and connection coefficients for perturbed Chebyshev polynomials

Zélia da Rocha (Faculty of Sciences of University of Porto, Portugal)

In this work we present the symbolic algorithm *PSDF* [2, 3] intended to explicit several semi-classical properties of perturbed second degree forms leading to the second order linear differential equation. We obtain results to the Chebyshev sequence of second kind for perturbations of several fixed orders [2, 3]. Furthermore we consider the problem of finding the connection coefficients [1] that allow to write perturbed Chebyshev polynomials of arbitrary order in terms of the sequence of second kind and in terms of the canonical basis.

References

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Yet another look at classical multiple d-orthogonal polynomials

Abdessadek Saib (University of Tebessa, Algeria)

We shall discuss in this talk new constructive characterisation for classical multiple d-orthogonal polynomials. Our arguments are based on the quasi-orthogonality's point of view. To some extent it is possible to construct the whole class of classical multiple d-orthogonal polynomials using only the concept of quasi-orthogonality together with the linear combination of polynomials. The above two concepts are fitted in together to show that classical multiple d-orthogonal polynomials could be characterised by some differential-difference equation as well as in terms of a specific linear combination. Applying the latter characterisations to some particular cases led to recover some classical, Δ_w as well as D_q-classical multiple d-orthogonal polynomials. Among other things, while working on the above and looking for multiple d-orthogonal polynomials as solutions of some differential-difference equations, a number of fascinating problems came across. We highlight some problems in this direction.

The growth of polynomials outside of a compact set – the Bernstein-Walsh inequality revisited

Klaus Schiefermayr (University of Applied Sciences Upper Austria, Wels, Austria)

Let K be a compact set in the complex plane $\mathbb{C}$ with logarithmic capacity $\operatorname{cap}(K) > 0$. Without loss of generality, we assume that K is such that $\text{Cov} \setminus K$ is connected, where $\text{Cov} := \mathbb{C} \cup \{\infty\}$. Let $g_K(z)$ denote the Green function (with pole at ∞) for $\text{Cov} \setminus K$, let P_n denote the set of all polynomials of degree n with complex coefficients and let $\|\cdot\|_K$ denote the supremum norm on K. Then the *Bernstein-Walsh inequality* says that for any polynomial $Q_n \in \mathrm{P}_n$,

$$\frac{|Q_n(z)|}{\|Q_n\|_K} \leq \exp(n \cdot g_K(z)) \qquad (z \in \mathbb{C} \setminus K).$$

The above inequality gives a very general upper bound for the modulus of a polynomial outside a compact set K with respect to its maximum value on K in terms of the corresponding Green function (which only depends on K). In this talk, we present some improvements of this inequality.

Computation of asymptotic expansions of turning point problems via Cauchy's integral formula

Javier Segura (Universidad de Cantabria, Spain)

Linear second order differential equations having a large real parameter and turning point in the complex plane are considered. Classical asymptotic expansions for solutions involve the Airy function and its derivative, along with two infinite series, the coefficients of which are usually difficult to compute. By considering the series as asymptotic expansions for two explicitly defined analytic functions, Cauchy's integral formula is employed to compute the coefficient functions to high order of accuracy. The method employs a certain exponential form of Liouville Green expansions for solutions of the differential equation, as well as for the Airy function. We illustrate the use of the method for the computation of Bessel functions and Laguerre polynomials of complex argument. This is joint work in collaboration with T.M. Dunster (San Diego State University, USA) and A. Gil (University of Cantabria, Spain).

Analytic computations of digamma function; a hypergeometric approach

Mohd Shadab (Jamia Millia Islamia, New Delhi, India)

With reference to high impact of interest in the investigation of polygamma functions, we have established some novel results for digamma function using Gauss, Lehmer, Jensen formulas, and hypergeometric approach. We have also presented some new summation theorems, and some modified summation theorems for Clausen's hypergeometric function.

Asymptotic properties of special functions of use in optical radiometry

Eric Shirley (NIST, Gaithersburg, USA)

As a first approximation, geometrical optics can be used to model classical radiometry, but the most accurate modeling of the flow of electromagnetic waves in optical systems usually relies on physical optics approximations based on Kirchhoff's Green's functions techniques. For short-wavelength or high-temperature blackbody sources, numerical treatments can be cumbersome and can be mitigated by studying the asymptotic properties of solutions. This work shall consider two classes of problems and demonstrate the utility of special functions in their solution. One class involves the treatment point and extended Lambertian sources, which relies on analysis using Bessel functions, Mellin transforms and distribution functions for the length of multiple-segment paths that traverse optical systems. The other class involves classical synchrotron radiation and computing the strengths of electric and magnetic fields, based on use of an oblate spheroidal coordinate system, Graf's addition theorem (the potential of which may not be fully appreciated), and Olver's uniform asymptotic expansion for Bessel functions in the transition region.

Non-classical orthogonal polynomials on the unit circle

Brian Simanek (Baylor University, USA)

In the theory of orthogonal polynomials on the unit circle (OPUC), many objects are in bijection with one another. These include non-trivial probability measures on the unit circle, non-trivial Schur functions, infinite CMV matrices, non-trivial Caratheodory functions, and sequences of complex numbers in the open unit disk (called Verblunsky coefficients). The core of the theory of OPUC is to understand the relationship between all of these objects. In this talk, we will demonstrate how to adapt certain aspects of the theory of OPUC to the situation when finitely many of the Verblunsky coefficients are outside the closed unit disk. Our main result will be an analog of Szegő's Theorem in this generalised setting. This is based on joint work with Maxim Derevyagin.

On the probability of positive-definiteness via semi-classical Laguerre polynomials

Nicholas Simm (University of Warwick, UK)

In this talk I will describe recent work on computing the probability that an $N \times N$ matrix from the generalised Gaussian Unitary Ensemble (gGUE) of random matrices is positive definite. This ensemble is like the GUE except that it contains a spectral singularity at the origin. We express the probability of interest in terms of the recurrence coefficients for a system of orthogonal polynomials known as semi-classical Laguerre polynomials. This system turns out to be closely related to the Painlevé IV differential equation. Then the large degree asymptotics of the polynomials and the relevant probabilities are computed using the steepest descent analysis of the corresponding Riemann-Hilbert problem. This is joint work with Alfredo Deaño (University of Kent, UK).

Fast and backward stable transforms between spherical harmonic expansions and bivariate Fourier series

Richard Mikael Slevinsky (University of Manitoba, Canada)

A rapid transformation is derived between spherical harmonic expansions and their analogues in a bivariate Fourier series. The change of basis is described in two steps: firstly, expansions in normalized associated Legendre functions of all orders are converted to those of order zero and one; then, these intermediate expressions are re-expanded in trigonometric form. The first step proceeds with a butterfly factorisation of the well-conditioned matrices of connection coefficients. The second step proceeds with fast orthogonal polynomial transforms via hierarchically off-diagonal low-rank matrix decompositions. Total pre-computation requires at best $\mathscr{O}(n^3 \log n)$ flops; and, asymptotically optimal execution time of $\mathscr{O}(n^2 \log^2 n)$ is rigorously proved via connection to Fourier integral operators.

Moment representations for exceptional orthogonal polynomial systems

Jessica Stewart Kelly (Christopher Newport University, Newport News, VA, USA)

It is well established that the classical orthogonal polynomial families outlined in Bochner's theorem can be viewed as the result of applying Gram-Schmidt to a particular sequence of moments. With some adaptations, this technique is applicable to the exceptional orthogonal polynomial systems of Laguerre and Jacobi, which occur when we study Sturm-Liouville type problems but allow for a finite number of degrees to be missing from the sequence of eigenpolynomials. This talk will discuss a universal result which relies on the moment functions and modified weights. Results for codimension one and two families will be presented along with a discussion of the difficulties that arise in higher codimension cases.

Spectral properties of block Jacobi matrices

Grzegorz Świderski (University of Wrocław, Poland)

Let $\mathscr{H}$ be a Hilbert space. Let us consider the recurrence relation

$$a_{n-1}^* u_{n-1} + b_n u_n + a_n u_{n+1} = z u_n, \quad (n > 0),$$

where $z \in \mathbb{C}$ and $(a_n : n \geq 0)$ and $(b_n : n \geq 0)$ are sequences of bounded operators on $\mathscr{H}$ such that for every n a_n is invertible and b_n is self-adjoint. The aim of the talk is to present asymptotics of the form

$$c_1(\|u_0\|^2 + \|u_1\|^2) \leq \|a_n\|(\|u_{n-1}\|^2 + \|u_n\|^2) \leq c_2(\|u_0\|^2 + \|u_1\|^2),$$

where constants $c_1, c_2 > 0$ are uniform with respect to z on compact subsets of $\mathbb{R}$ (or $\mathbb{C}$). As a consequence we present simple criteria for the complete indeterminate case for operator-valued orthogonal polynomials. They are new even for $\mathscr{H} = \mathbb{C}^d$ with $d > 1$.

Generalised Poisson–Mehler summation formula as a source for bivariate orthogonal polynomials on $[-1,1] \times [-1,1]$

Paweł J. Szabłowski (Warsaw University of Technology, Poland)

We study polynomials in x and y of degree $n+m : \{Q_{m,n}(x,y|t,q)\}_{n,m\geq 0}$ that are related to the generalisation of Poisson–Mehler formula i.e. to the expansion

$$\sum_{i\geq 0} \frac{t^i}{[i]_q!} H_{i+n}(x|q) H_{m+i}(y|q) = Q_{n,m}(x,y|t,q) \sum_{i\geq 0} \frac{t^i}{[i]_q!} H_i(x|q) H_m(y|q),$$

where $\{H_n(x|q)\}_{n\geq -1}$ are the so-called q-Hermite polynomials (qH). In particular we show that the spaces

$$span\{Q_{i,n-i}(x,y|t,q) : i = 0,\ldots,n\}_{n\geq 0}$$

are orthogonal with respect to a certain measure (two-dimensional $(t,q)-$Normal distribution) on the square

$$\left\{(x,y) : |x|,|y| \leq 2/\sqrt{1-q}\right\}$$

being a generalisation of two-dimensional Gaussian measure. We study structure of these polynomials showing in particular that they are rational functions of parameters t and q. We use them in various infinite expansions that can be viewed as simple generalisation of the Poisson-Mehler summation formula. Further we use them in the expansion of the reciprocal of the right hand side of the Poisson-Mehler formula.

Leonard triples of q-Racah type and their pseudo intertwiners

Paul Terwilliger (University of Wisconsin, Madison, USA)

Let $\mathbb{F}$ denote a field, and let V denote a vector space over $\mathbb{F}$ with finite positive dimension. Pick a nonzero $q \in \mathbb{F}$ such that $q^4 \neq 1$, and let A, B, C denote a Leonard triple on V that has q-Racah type. We show that there exist invertible W, W', W'' in $\mathrm{End}(V)$ such that (i) A commutes with W and $W^{-1}BW - C$; (ii) B commutes with W' and $(W')^{-1}CW' - A$; (iii) C commutes with W'' and $(W'')^{-1}AW'' - B$. Moreover each of W, W', W'' is unique up to multiplication by a nonzero scalar in $\mathbb{F}$. We show that the three elements $W'W, W''W', WW''$ mutually commute, and their product is a scalar multiple of the identity. A number of related results are obtained. We call W, W', W'' the *pseudo intertwiners* for A, B, C.

Some determinant formulas for ratio of two series

Mikhail Tyaglov (Shanghai Jiao Tong University, China)

Some well-known and possibly new determinant formulas for rational functions and ratios of two formal power series are presented. We discuss applications of these formulas for determinant representation of moments for certain measures and for study parametric behaviour of moments.

A model for the higher rank Racah algebra

Wouter van de Vijver (Ghent University, Belgium)

We propose a generalisation of the Racah algebra by considering the tensor product of n copies of $su(1,1)$. Its role as symmetry algebra for the Z_n^2 Dunkl-Laplacian and its connection to the generalised Bannai-Ito algebra are explained. Bases for Dunkl-harmonics are constructed so that each basis consists of joint eigenfunctions of a maximal abelian subalgebra of the generalised Racah algebra. A method is provided for finding the connection coefficients between these bases. The connection coefficients correspond to the multivariate Racah polynomials as defined by M.V. Tratnik. We also propose a realisation of the higher rank Racah algebra in terms of shift operators. This realisation contains the operators found by J. Geronimo and P. Iliev which have the multivariate Racah polynomials as eigenfunctions. This is joint work with Hendrik De Bie, Vincent Genest and Luc Vinet.

Classical q-orthogonal polynomials that have simple sequences of moments

Luis Verde-Star (Universidad Autonoma Metropolitana, Mexico)

We obtain a three-term recurrence relation with variable coefficients for the sequences of normalized moments ($\mu_0 = 1$) of all the classical q-orthogonal polynomials. The coefficients of the recurrence relation depend on the coefficients of the q-difference equation satisfied by the polynomials. Considering the cases in which the recurrence relation simplifies, we obtain some families of classical q-orthogonal polynomials that have simple explicit formulas for the moments, such as, $\mu_n = 1/[n+1], \mu_n = 1/[n]!, \mu_n = [n]!$, and also $\mu_n = [r][r+1][r+2]\cdots[r+n-1]$ and $\mu_{2n} = 1/[2n+1]$ and $\mu_{2n+1} = 0$. For each case we find explicit expressions for the coefficients of the three-term recurrence relation and for the coefficients of the q-difference equation satisfied by the polynomial sequence. The polynomial sequences with simple moments belong to the families of q-Krawtchouk polynomials with negative parameter N, q-Bessel polynomials, little q-Laguerre/Wall polynomials, and big q-Jacobi polynomials. (Joint work with M.I. Arenas-Herrera).

Tridiagonal representations of the q-oscillator algebra and Askey-Wilson polynomials

Luc Vinet (Université de Montréal, Canada)

A construction is given of the representations of the q-oscillator algebra where both generators are tridiagonal. It is shown to be connected to the Askey-Wilson polynomials. Based on work with S. Tsujimoto (Kyoto Uinversity) and A. Zhedanov (Renmin Uinversity)

Approximation by Durrmeyer type Jakimovski-Leviatan operators involving Brenke polynomials

Shahid Ahmad Wani (Aligarh Muslim University, India)

This talk aims to introduce the Durrmeyer type Jakimovski-Leviatan operators involving Brenke type polynomials. The positive linear operators including the Brenke polynomials sequence are constructed and their approximation properties are established. Further, the convergence properties and the order of convergence of these operators in a weighted space of functions are considered. Furthermore, the Voronovskaja type theorem is illustrated for the operators including a special case of Brenke type polynomials.

Integral transforms of generalised k-Mittag-Leffler function

Shorab Wali Khan (Aligarh Muslim University, India)

Remarkably, large number of integral formulas involving a variety of special functions have been developed by many authors. Also many integral formulas involving Mittag-Leffler function have been exhibited. In this paper, we establish two new integral formulas involving the generalised k-Mittag-Leffler function, which are expressed in terms of the generalised (Wright) hypergeometric functions.

Spectra of Jacobi operators via connection coefficient matrices

Marcus Webb (KU Leuven, Belgium)

Between two families of orthogonal polynomials there is a triangular matrix called the connection coefficient matrix, which encodes the change of basis between the two bases. It is directly computable from the three term recurrences. We show that the connection coefficients also encode, in a very explicit way, relationships between their weights of orthogonality, which are the spectral measures of their Jacobi operators. This leads to new methods of computing the spectral measure for orthogonal polynomials with perturbed three term recurrences. The talk is based on arxiv.org/abs/1702.03095 which is joint work with Sheehan Olver (Imperial College, London, UK).

Spectral approximation of convolution operator

Kuan Xu (University of Kent, UK)

Convolution operator is ubiquitously dense in mathematics and engineering. While approximations via classic orthogonal polynomials for many commonly-used operators, e.g. integration and differentiation, are well-known for decades and become indispensable in approximation theory and spectral methods, spectral approximation of convolution operator hasn't been attempted until very recent years. In this talk, some recent results on the convolution of classic orthogonal polynomials and spectral approximations of convolution operator will be presented. These results enable accurate computation of convolution integrals and are believed to lay the foundation of the spectral methods for convolution integral equations.

Positive definite functions on the unit sphere and integral of Jacobi polynomials

Yuan Xu (University of Oregon, Eugene, USA)

We establish the positivity of a family of integrals of Jacobi polynomials, which shows, in particular, that, for each $0 < t < \pi$, the function $\theta \mapsto (t-\theta)_+^\delta$ is positive definite on the unit sphere S^{d-1} if $\delta \geq d/2$ for $d = 3, 4, \ldots$, proving a conjecture of R. Beatson, W. zu Castel and the author.

On certain identities, connection and explicit formulas for the Bernoulli, Euler numbers and Riemann's zeta-values

Semyon Yakubovich (University of Porto, Portugal)

Various new identities, recurrence relations, integral representations, connection and explicit formulas are established for the Bernoulli, Euler numbers and the values of Riemann's zeta function. To do this, we explore properties of some Sheffer's sequences of polynomials related to the Kontorovich-Lebedev transform.

Inverse results on row sequences of Hermite-Padé approximation

Yanely Zaldivar Gerpe (Universidad Carlos III de Madrid, Spain)

We study inverse type results for incomplete Padé approximants of analytic functions under the assumption that the sequence of denominators of the approximating rational functions have limit. Such results allow to describe the region to which the analytic function can be extended meromorphically, determine the location and order of the poles in this region, and detect some singularities on the boundary. These results are applied to the study of Hermite-Padé approximants; that is interpolating vector rational functions of vectors of analytic functions.

Posters

Convexity/concavity properties and asymptotics of the median of the beta distribution

Dimitris Askitis (University of Copenhagen, Denmark)

The median of the beta distribution is defined implicitly by the equation $\int_0^q t^{a-1}(1-t)^{b-1}\mathrm{d}t = B(a,b)/2$. We study the median as a univariate function of one of its parameters, considering the other one constant. We study monotonicity and convexity/concavity properties of it and of its logarithm. We find the corresponding asymptotic expansions at 0 and infinity.

Algebraic equation and symmetric second degree forms of class two.

Belkis Bel Haj Ali (University El Manar, Tunis, Tunisia)

We study the following problem: Let v be a symmetric regular form. Find an explicit necessary and sufficient conditions for the regularity of the forms u satisfying: $Ru = hv$; with h non zero and $R(x) = x^2 - c^2$; and $c \geq 0$. Also, the coefficients of the second-order recurrence relation satisfied by the monic orthogonal polynomial sequence (MOPS) with respect to u are obtained. and we reveal some results concerning the semi-classical character and the class of the form u when v is supposed semi-classical of class s and we establish that if v is a second degree form, then u is also a second degree form. Finally we give a connection between some symmetric second degree forms of class two and the Chebyshev form of the second kind. This allows an exhaustive study of a class of second degree forms of class 2.

Some partitions of polynomial sets

Youssef Ben Cheikh (Monastir University, Tunisia)

Let $\mathscr{P}$ be the set of polynomial sequences $\{P_n\}_{n\geq 0}$ where $P_n \in \mathscr{C}[X]$ and degree$(P_n) = n$. In this work, we consider three sets of operators acting on formal power series and we show that any partition of these sets leads to a partition of $\mathscr{P}$. Then we define suitable partitions of these sets to obtain the origins of some well known classes in $\mathscr{P}$. We propose some models for the description and we discuss some applications.

Strict positive definiteness of product covariance functions on manifolds

Rafaela Bonfim (São Paulo University, Brazil)

Let X denote a compact two-point homogeneous space. Given two continuous and isotropic positive definite kernels on X, we determine necessary and sufficient conditions in order that their product be strictly positive definite on X. We also consider the very same question in the space-time setting, that is, the case X is the cartesian product of a locally compact group G and the d dimensional unit sphere and both kernels are continuous and isotropic with respect to the sphere component.

Connection and linearization coefficients for basic hypergeometric polynomials.

Hamza Chaggara (Sousse University, Tunisia)

In this work we give a closed-form expression of the inversion, the connection and the linearization coefficients for general basic hypergeometric polynomial sets using some known inverse relations. We derive expansion formulae corresponding to all the families within the q-Askey scheme.

Ratio asymptotic for matrix functions of second kind

Juan Carlos García Ardila (University Carlos III de Madrid, Spain)

We have studied the interaction between matrix orthonormal polynomials (with respect to a matrix of measures definite positive), matrix associated polynomials and matrix functions of second kind given various formulas in terms of quasi-determinants. Here, we emphasise on the asymptotic behaviour of the ratio between two consecutive second kind functions.

Neural system for determining sequences of orthogonal polynomials

Eva María García del Toro (Universidad Politécnica de Madrid, Spain)

This work proposes a neural system for determining the sequences of coefficients of the three-term recurrence relation associated with a sequence of orthogonal polynomials. The proposed approach has two steps: First, a novel neural architecture calculates the first terms of the sequences of coefficients. Then, an evolutionary algorithm autonomously constructs full-connected multilayered feedforward neural architectures to approximate the next terms. The evolutionary algorithm employs a novel context-free grammar with three significant properties: in the first place, all sentences belonging to the language produced by the grammar only encode valid neural architectures. Secondly, a full connected feedforward neural architectures of any size can be generated. Finally, in order to avoid overfitting, smallest neural architectures are firstly chosen. The proposed method has been aplied to several recurrence relations associated with classical orthogonal polynomials such as Laguerre, Legendre and others, obtaining very satisfactory results. This is a joint work with D. Barrios Rolanía, G. Delgado Martínez and D. Manrique

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Generalised Freud polynomials and the Painlevé equations

Abey Kelil (University of Pretoria, South Africa)

We investigate certain properties of monic polynomials orthogonal with respect to a semi-classical generalised Freud weight $w_\lambda(x;t) = |x|^{2\lambda+1}\exp(-x^4+tx^2)$, where $x \in \mathbb{R}$ and the parameters $\lambda > 0$ and $t \in \mathbb{R}$. generalised Freud polynomials arise from a symmetrization of semi-classical Laguerre polynomials given in [1]. We prove that the coefficients in the three-term recurrence relation associated with the generalised Freud weight $w_\lambda(x;t)$ can be expressed in terms of Wronskians of parabolic cylinder functions that appear in the description of special function solutions of the fourth Painlevé equation. This closed form expression for the recurrence coefficients allows the investigation of certain properties of the generalised Freud polynomials. We obtain an explicit formulation for the generalised Freud polynomials in terms of the recurrence coefficients, investigate the higher order moments as well as the Pearson equation satisfied by the generalised Freud weight $w_\lambda(x;t)$. We also derive a differential-difference equation using a method due to Shohat and second-order linear ordinary differential equation satisfied by the polynomials. Further, we provide an extension of Freud's conjecture for the recurrence coefficient $\beta_n(t;\lambda)$ associated with the generalised Freud weight. In particular, following the works of G. Freud (cf. [2, 3, 4]), we provide the asymptotic behaviour of the recurrence coefficient $\beta_n(t;\lambda)$ as the degree, or alternatively, the parameter tends to infinity. Keywords: Semi-classical orthogonal polynomials, generalised Freud weight, recurrence coefficient, Painlevé equations.

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Bispectral Darboux transformations of the Darboux-Pöchl-Teller operator

Abdul Muqeet Khalid (University of Leeds, UK)

The bispectral problem was first posed by Duistermaat and Grünbaum in 1986, who went on to solve it in the case of a one-dimensional Schrödinger operator. Since then, much progress has been made in the derivation and analysis of various bispectral families, revealing interesting links with nonlinear integrable PDEs, algebraic geometry, orthogonal polynomials and special functions. Bispectral operators of rank one have been completely classified by G. Wilson who showed that they correspond to rational solutions to the KP equation. For higher ranks, the classification problem remains open, but some large bispectral families have been found in relation to Bessel and Airy functions in the works of Bakalov, Horozov, Yakimov, Kasman and Rothstein. If one generalises the bispectral problem by allowing difference operators in the spectral variable, then this has a clear parallel with the three-term recurrence relation in the theory of orthogonal polynomials. This differential-difference version of the bispectral problem has also been studied extensively, more recently in the context of the exceptional orthogonal polynomials. In most of these works, Darboux transformations played a key role. However, the associated special functions have not been amenable to such a treatment, until now. In our work, we make a step in that direction by constructing a large family of bispectral operators, analogous to the Bakalov-Horozov-Yakimov family, but based on the Jacobi function instead of the Bessel function. Our construction uses in an essential way the properties of the monodromy group of the hypergeometric equation. This is joint work with Oleg Chalykh.

On the classical character of Sheffer-Meixner 2-orthogonal polynomials type

Ali Krelifa (University of Khemis Miliana, Algeria)

In this paper we consider a generalisation of the problem considered in [P. Maroni and J. Van Iseghem, Generating functions and semi-classical orthogonal polynomials, *Proc. Roy. Soc. Edinburgh Sect. A*, **124** (1994) 1003–1011]. The aim of this generalisation is to look at the classical character of the so called Sheffer-Meixner 2-orthogonal polynomials type.

Asymptotic relations between 2-orthogonal polynomials

Imed Lamiri (University of Sousse, Tunisia)

In this work, we consider a natural extension, in the context of d-orthogonality, for asymptotic analysis of the Askey-scheme. We point out several 2-orthogonal polynomials which admit asymptotic expansions in terms of Gould-Hoper polynomials. From those expansions several limits between 2-orthogonal polynomials are obtained.

Asymptotic behaviour of eigenvalues of a differential operator for Gegenbauer-Sobolev orthonormal polynomials

Juan Francisco Mañas-Mañas (Universidad de Almería, Spain)

We consider the Sobolev inner product

$$(f,g)_S := \int_{-1}^{1} f(x)g(x)(1-x^2)^{\alpha}\,\mathrm{d}x + M\big[f^{(j)}(-1)g^{(j)}(-1) + f^{(j)}(1)g^{(j)}(1)\big],$$

where $\alpha > -1$, $j \in \mathbb{N}\cup\{0\}$ and $M > 0$. We denote by $\{Q_n^{(\alpha,j,M)}\}_{n\geq 0}$ the sequence of orthonormal polynomials with respect to this inner product. These polynomials are called Gegenbauer–Sobolev orthogonal polynomials since they involve the classical Gegenbauer weight. In [1] the authors give conditions to ensure that there is a differential operator $\mathbf{T}$, such that the polynomials $Q_n^{(\alpha,j,M)}$ are eigenfunctions of $\mathbf{T}$ and λ_n are the corresponding eigenvalues, i.e.

$$\mathbf{T}Q_n^{(\alpha,j,M)}(x) = \lambda_n Q_n^{(\alpha,j,M)}(x).$$

We are interested in the asymptotic behaviour of the eigenvalues λ_n, which will be useful to compute

$$r_0 := \lim_{n\to+\infty} \frac{\log(\max_{x\in[-1,1]} |Q_n^{(\alpha,j,M)}(x)|)}{\log(\lambda_n)}.$$

This value r_0 is related to the convergence of a series in a left–definite space. Furthermore, we study the Mehler–Heine asymptotics for the polynomials $Q_n^{(\alpha,j,M)}$. Joint work with Lance L. Littlejohn (Baylor University, USA), Juan J. Moreno-Balcázar (Universidad de Almería, Spain) and Richard Wellman (Westminster College, USA) (see [2]).

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Fast, reliable and unrestricted computation of classical Gaussian quadratures

Diego Ruiz-Antolín (University of Cantabria, Spain)

Gaussian quadrature rules are one of the most fundamental methods for numerical integration, and particularly the classical quadrature rules (Hermite, Laguerre, Jacobi). For moderate orders, a simple and well known method to compute Gaussian quadratures is provided by the Golub-Welsch algorithm, based on the diagonalization of the associated Jacobi matrix. However, for high orders the Golub-Welsch method is inefficient because the complexity scales quadratically with the order, and iterative methods become preferable. We describe iterative methods of computation of classical Gaussian quadrature rules which are effective both for small and large degree and which are fast and reliable in the sense that the iterative computation of the Gaussian nodes has guaranteed convergence of order 4. Additionally, the methods are practically unrestricted with respect to the ranges of parameters available and they are asymptotically exact as the order becomes large. The fast and certain convergence of the iterative method makes it an ideal approach for high precision computations. This is joint work in collaboration with A. Gil (University of Cantabria, Spain), J. Segura (University of Cantabria, Spain) and N.M. Temme (CWI, The Netherlands).
