

MATHEMATICS & STATISTICS/ A-LEVEL CORE FORMULA SHEET



School of
Mathematics, Statistics
and Actuarial Science

Algebra, Geometry and Functions

Quadratic Equation

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Logs and Exponentials

$$y = b^x \Leftrightarrow x = \log_b(y), \text{ for } b, y > 0$$

$$\log_b(p) + \log_b(q) = \log_b(pq)$$

$$\log_b(p) - \log_b(q) = \log_b(p/q)$$

$$\log_b(p^k) = k \log_b(p)$$

$$b^{\log_b(x)} = x, \quad \log_b(b^x) = x$$

$$\ln(x) = \log_e(x), \quad e^{\ln(x)} = \ln(e^x) = x$$

Odd and Even Functions

$$f(-x) = -f(x) \Leftrightarrow f(x) \text{ is odd}$$

$$f(-x) = f(x) \Leftrightarrow f(x) \text{ is even}$$

Straight Lines

Line with gradient m through (x_1, y_1) has equation $(y - y_1) = m(x - x_1)$

Lines with gradients m_1 and m_2 are perpendicular if $m_1 m_2 = -1$

Circles

Circle with centre $C(a, b)$ and radius r has equation $(x - a)^2 + (y - b)^2 = r^2$

Sequences and Series

Arithmetic Sequences

For an arithmetic sequence with first term a , last term l , and common difference d :

$$\text{nth term} = u_n = a + (n - 1)d$$

$$\text{Sum to } n \text{ terms} = S_n = \frac{1}{2}n(2a + (n - 1)d) = \frac{1}{2}n(a + l)$$

Geometric Sequences

For a geometric sequence with first term a and common ratio r :

$$\text{nth term} = u_n = ar^{n-1}$$

$$\text{Sum to } n \text{ terms} = S_n = \frac{a(1 - r^n)}{1 - r}, \text{ for } r \neq 1$$

$$\text{Sum to infinity} = S_\infty = \frac{a}{1 - r}, \text{ for } |r| < 1$$

Binomial Series

$$\text{Binomial coefficient is } {}^n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

For $n \in \mathbb{N}$,

$$(a + b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n$$

For $n \in \mathbb{R}$ and $|b| < |a|$,

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \dots + \frac{n(n-1)\cdots(n-r+1)}{r!}a^{n-r}b^r + \dots$$

Numerical Methods

Trapezium Rule

$$\int_a^b y \, dx \approx \frac{h}{2}(y_0 + y_n) + h(y_1 + y_2 + \dots + y_{n-1}),$$

where $h = \frac{b - a}{n}$, $x_k = a + kh$, $x_n = b$, and $y_k = f(x_k)$

Newton-Raphson Iteration

$$\text{To solve } f(x) = 0, \text{ use } x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Trigonometry

Radians

$$2\pi \text{ radians} = 360^\circ$$

For a sector of angle θ radians in a circle of radius r :

$$\text{Arc length} = s = \theta r$$

$$\text{Sector area} = A = \frac{1}{2}\theta r^2$$

Triangles

$$\text{Sine rule: } \frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

$$\text{Cosine rule: } a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$\text{Area} = \frac{1}{2}ab \sin(C)$$

Trig Identities

Pythagorean Identities:

$$\sin^2(\theta) + \cos^2(\theta) \equiv 1$$

$$\tan^2(\theta) + 1 \equiv \sec^2(\theta)$$

$$1 + \cot^2(\theta) \equiv \operatorname{cosec}^2(\theta)$$

Sum/Difference Identities

$$\sin(a \pm b) = \sin(a) \cos(b) \pm \sin(b) \cos(a)$$

$$\cos(a \pm b) = \cos(a) \cos(b) \mp \sin(a) \sin(b)$$

$$\tan(a \pm b) = \frac{\tan(a) \pm \tan(b)}{1 \mp \tan(a) \tan(b)}$$

Double Angle Formulae

$$\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\tan(2\theta) = \frac{2 \tan(\theta)}{1 - 2 \tan(\theta)}$$

Small Angle Approximations

When θ (in radians) is small: $\sin(\theta) \approx \theta, \cos(\theta) \approx 1 - \frac{1}{2}\theta^2, \tan(\theta) \approx \theta$

Calculus

Table of Derivatives

Function	Derivative
x^n	nx^{n-1}
a^{kx}	$k \ln(a)a^{kx}$
e^{kx}	ke^{kx}
$\ln(kx)$	$\frac{1}{x}$
$\sin(kx)$	$k \cos(kx)$
$\cos(kx)$	$-k \sin(kx)$
$\tan(kx)$	$k \sec^2(kx)$
$\sec(kx)$	$k \sec(kx) \tan(kx)$
$\cosec(kx)$	$-k \cosec(kx) \cot(kx)$
$\cot(kx)$	$-k \cosec^2(kx)$

Rules

Differentiation from first principles:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Chain rule:

$$y = f(g(x)) \Rightarrow \frac{dy}{dx} = f'(g(x)) \times g'(x)$$

$$\text{or } y = f(g(x)) = f(u) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Product rule:

$$y = uv \Rightarrow \frac{dy}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$$

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Quotient rule:

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$$

Inverse rule:

$$1 / \frac{dy}{dx} = \frac{dx}{dy}$$

Parametric rule: For a curve given by $x = f_1(t)$ and $y = f_2(t)$,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Table of Integrals

Function	Integral (remember to add +C!)
x^n	$\frac{x^{n+1}}{n+1} \quad (n \neq -1)$
$x^{-1} = \frac{1}{x}$	$\ln(x)$
e^{kx}	$\frac{1}{k}e^{kx}$
$\sin(kx)$	$-\frac{1}{k} \cos(kx)$
$\cos(kx)$	$\frac{1}{k} \sin(kx)$
$\tan(kx)$	$\frac{1}{k} \ln \sec(kx) $
$\sec^2(kx)$	$\frac{1}{k} \tan(kx)$
$\cosec^2(kx)$	$-\frac{1}{k} \cot(kx)$
$\cot(kx)$	$\frac{1}{k} \ln \sin(kx) $
$\frac{f'(x)}{f(x)}$	$\ln f(x) $

Rules

Definite integration: $\int_a^b f(x) dx = F(b) - F(a)$, where $F(x)$ is the integral of $f(x)$

Integration by parts: $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Mechanics

Forces

Weight = mg

Friction: $F \leq \mu R$

Newton's 2nd law: $F = ma$

Kinematics

For 1D motion with constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

For 1D motion with variable acceleration:

r (position), v , and a are all functions of t ; the above suvat equations no longer apply, so use

$$v = \frac{dr}{dt}, \quad a = \frac{dv}{dt}, \quad r = \int v dt, \quad v = \int a dt$$

Statistics

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ for } P(B) \neq 0$$

Summary Statistics

$$\text{Mean} = \bar{x} = \frac{\sum x_i}{n} = \frac{\sum f_i x_i}{\sum f_i}, \quad \text{Variance} = \sigma^2$$

$$\text{Standard dev.} = \sigma = \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2} = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \bar{x}^2}$$

Outliers are any data outside of the interval $\bar{x} \pm 2\sigma$ or $Q_1 - 1.5\text{IQR}$ and $Q_3 + 1.5\text{IQR}$

Binomial Distribution

If $X \sim B(n, p)$, then $P(X = r) = {}^n C_r p^r (1-p)^{n-r}$

Mean of $X = np$, variance of $X = np(1-p)$

Normal Distribution

If $X \sim N(\mu, \sigma^2)$, then $Z = \frac{X - \mu}{\sigma}$ with $Z \sim N(0, 1)$

Hypothesis test for the mean: if $X \sim N(\mu, \sigma^2)$, then

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \quad \text{and} \quad \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$