CAPTURE-RECAPTURE MODELS IN ECOLOGY: MULTI-STATE DEVELOPMENTS

RACHEL MCCREA

GERMAN STATISTICAL WEEK

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COLLABORATORS

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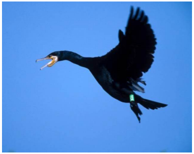
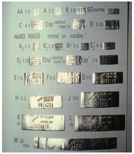
- Ruth King
- Richard Griffiths

Outline

Individual marking





Capture-recapture data

- 1 0 0 1 0
- 1 1 0 1 1
- 0 0 1 0 1
- ...

Closed population model, M_t

- p_t: probability an individual is captured at occasion t.
- Capture-recapture data and probabilities
  - `1 0 0 1 0` $p_1(1-p_2)(1-p_3)p_4(1-p_5)$
  - `1 1 0 1 1` $p_1 p_2 (1-p_3) p_4 p_5$
  - `0 0 1 0 1` $(1-p_1)(1-p_2)p_3(1-p_4)p_5$
  - ...

CLOSED POPULATION MODEL

- Some individuals will not be captured at all during the study;
- The encounter history for these individuals is given by
  - 0 0 0 0 0 $(1-p_1)(1-p_2)(1-p_3)(1-p_4)(1-p_5)$
- It is the number of individuals who are never captured that we need to estimate.

The likelihood has the form:

$$L \propto \frac{N!}{(N - D!)} \prod_{i=1}^{D} \Pr(h_i) \times \Pr(h_0)^{N-D} \tag{1}$$

- h_i: observed encounter history for individual i;
- h_i: observed encounter history of never encountered;
- N: population size;
- D: number of observed individuals.

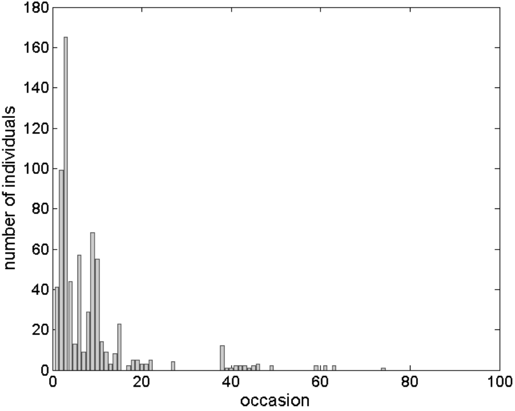
Closed population model, M_b

- p: probability of initial capture;
- c: probability of subsequent capture.

- Capture-recapture data and probabilities
  - 1 0 0 1 0 $p(1-c)(1-c)c(1-c)$
  - 1 1 0 1 1 $pc(1-c)cc$
  - 0 0 1 0 1 $(1-p)(1-p)p(1-c)c$
  - ...

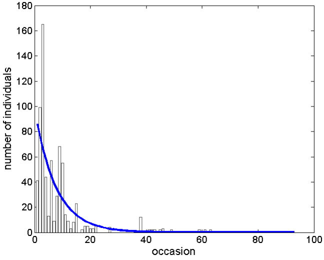
REMOVAL DATA

n_t: size of sample removed at sample t.



LINK TO MODEL M_b

- Basic geometric model
  Pr(individual is removed at occasion t) $= (1-p)^{t-1}p$
- Same model as used for time to conception for human couples;
- Equivalent to estimating p in M_b, and assuming $c = 0$.



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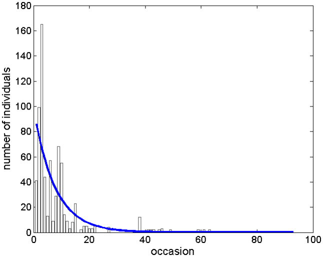
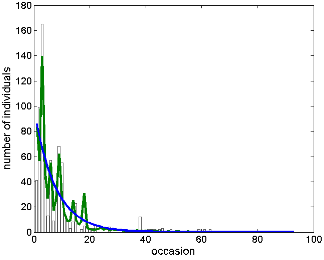
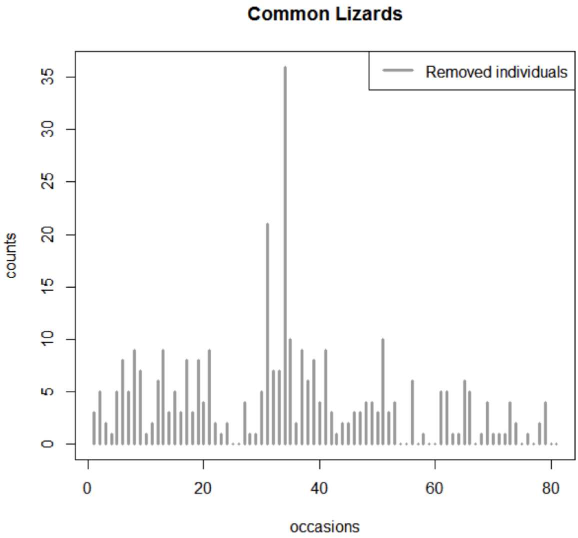
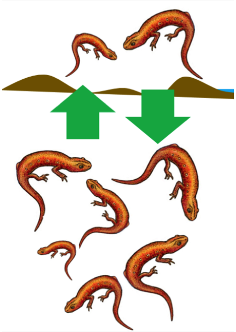




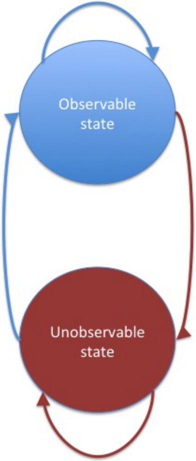
Why do data exhibit unexpected peaks?



AN UNDERGROUND CITY?

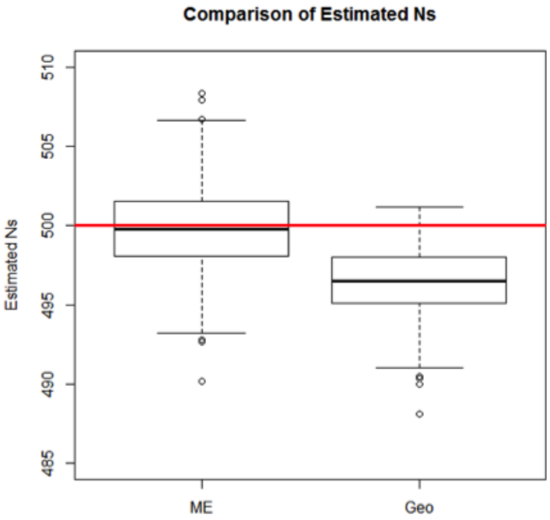


Multistate removal model

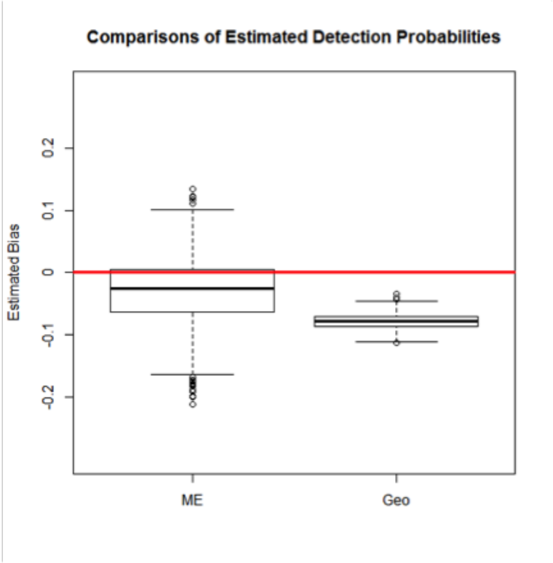


- Develop a two-state model, with one unobservable state with capture probability of 0;
- Naturally fits into a multievent framework, which is an HMM.

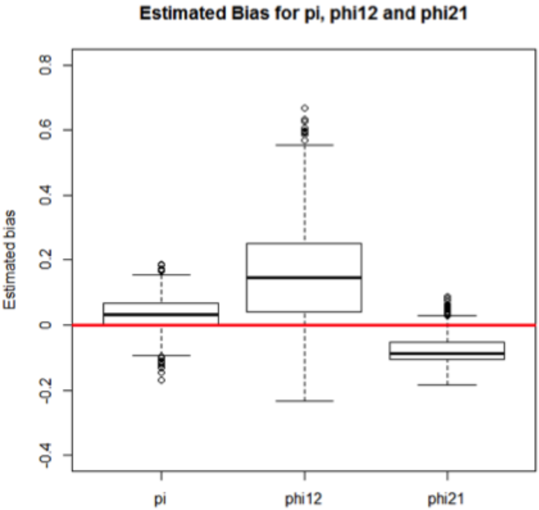
Simulation results



SIMULATION RESULTS



SIMULATION RESULTS



PARAMETER REDUNDANCY

- A model is parameter redundant if you cannot estimate all of the parameters;
- Parameter redundancy is diagnosed by forming a derivative matrix $D = \partial\kappa/\partial\theta$ where κ denotes an exhaustive summary for a model that provides a unique representation of the model and θ denotes the parameters;
- If $\text{rank}(D) = \dim(\theta)$, all parameters are estimable;
- If $\text{rank}(D) < \dim(\theta)$ the model is parameter redundant.

PARAMETER REDUNDANCY

- Model $\pi, p, \psi_{12}, \psi_{21}$ is parameter redundant;
- The estimable parameters are: πp, $p\psi_{21}$ and $p(\psi_{12} - 1) - \psi_{12} - \psi_{21}$.
- If p is modelled using a temporal covariate, the model is full rank.

Jolly-Seber model

- The studied population might not be closed, but still want to be able to estimate population size;
- Parameters for the Jolly-Seber model:
  - N: population size;
  - β_t : proportion of individuals first available for capture at occasion t+1;
  - p_t: probability an individual is captured at occasion t;
  - ϕ_t: probability an individual present in the study area at occasion t remains in the study area until occasion t+1.

JOLLY-SEBER MODEL

- When forming the probability of an observed encounter history we need to sum over possible entry and departure times.
  - Suppose individual i is first captured at occasion f_i and last captured at occasion l_i;
  - $x_{ij} = 1$ if individual i is captured at occasion j, $x_{ij} = 0$ otherwise.

$$\Pr(h_i) = \sum_{b=1}^{f_i} \sum_{d=l_i}^{T} \beta_{b-1} \left(\prod_{j=b}^{d-1} \phi_j \right) (1 - \phi_d) \left\{ \prod_{j=b}^{d} p_j^{x_{ij}} (1 - p_j)^{1-x_{ij}} \right\}$$

Jolly-Seber model

Corresponding probability of an individual not captured during the study:

$$\Pr(h_0) = \sum_{b=1}^{T} \sum_{d=1}^{T} \beta_{b-1} \left(\prod_{j=b}^{d-1} \phi_j \right) (1 - \phi_d) \left\{ \prod_{j=b}^{d} (1 - p_j) \right\}$$

The likelihood, once again, has the same form:

$$L \propto \frac{N!}{(N - D!)} \prod_{i=1}^{D} \Pr(h_i) \times \Pr(h_0)^{N-D} \quad (1)$$

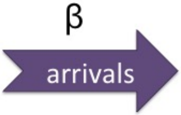
STOPOVER MODEL

- Generalised version of the Jolly-Seber model (Pledger et al, 2009)
- Parameters are defined to be age-dependent, where **age** corresponds to the time spent in study area:
  - N: population size;
  - β_t : proportion of individuals first available for capture at occasion t+1;
  - $p_t(a)$: probability an individual which entered the study a occasions previously is captured at occasion t;
  - $\phi_t(a)$: probability an individual present in the study area at occasion t, which entered the study a occasions previously, remains in the study area until occasion t+1.
- Can naturally be expressed in an HMM framework.

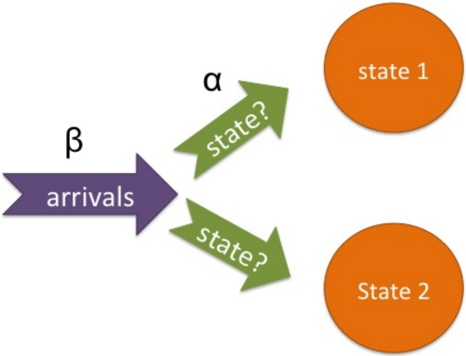
Multistate stopover model

- Individuals may be captured in different states;
- Multistate extensions exist for many capture-recapture models;
- Demonstrate that its possible to build transitions and state-dependence into the basic stopover model;
- HMM provides a useful, efficient framework for this.

Multistate stopover model



MULTISTATE STOPOVER MODEL



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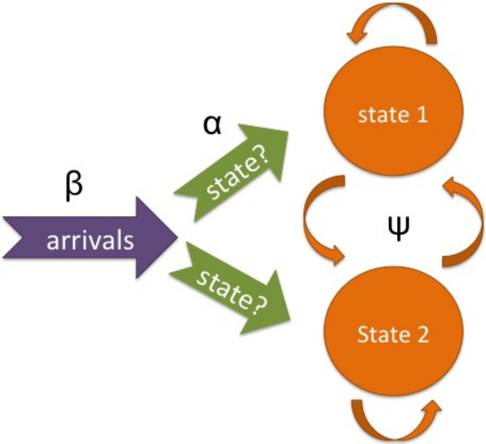
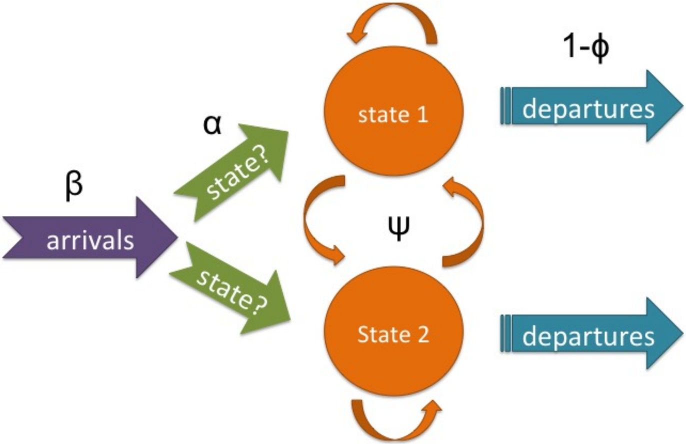
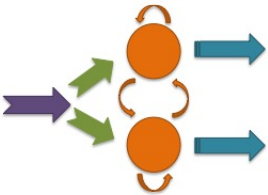
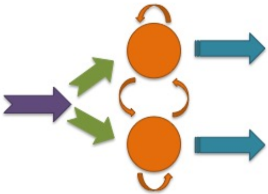


MULTISTATE STOPOVER MODEL

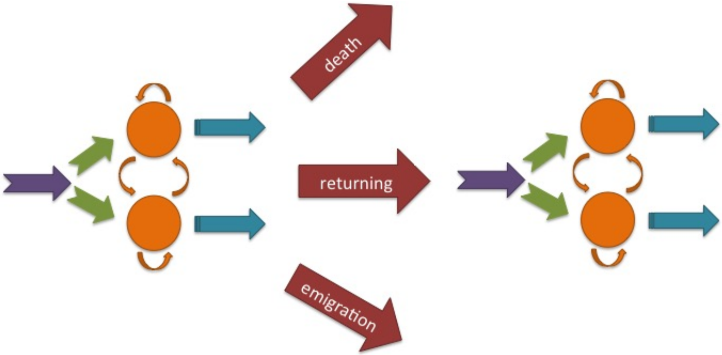


Integrating over multiple years





INTEGRATING OVER MULTIPLE YEARS



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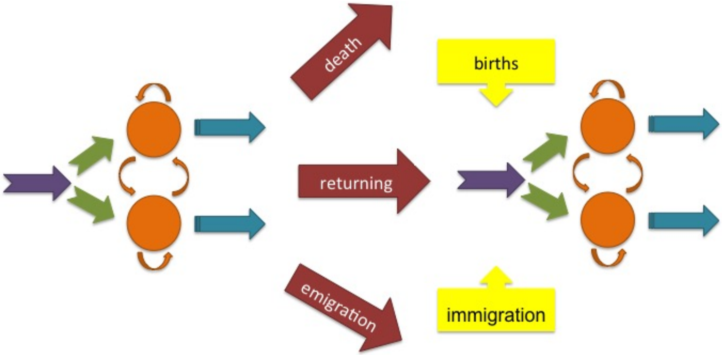
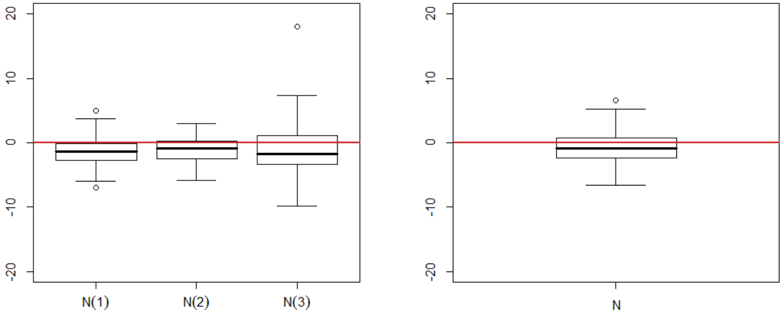


SIMULATION RESULTS



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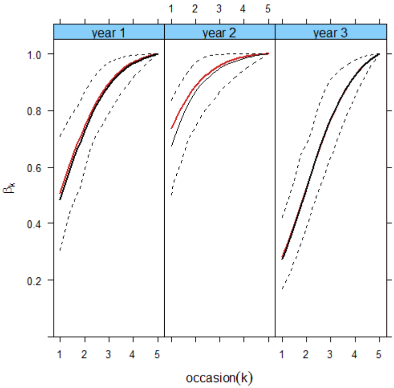
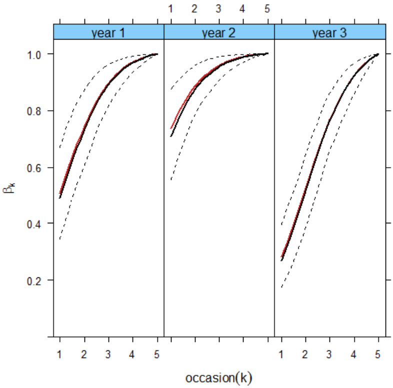
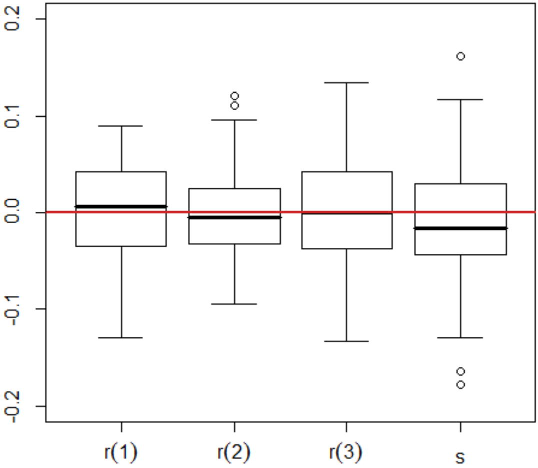




Simulation results



ADVANTAGES

- General framework, with other models forming a special case;
  - Robust design (closed and open);
  - Closed population models - including a multistate closed population model (Worthington et al, 2015);
  - Stopover and Jolly-Seber models;
- Using all available data in a coherent model - compare Besbeas et al (2002);
- Natural generalisation of model selection methods for multistate models
  - Transdimensional simulated annealing (Brooks et al, 2003);
  - Step-wise procedures using score tests (McCrea and Morgan, 2011);

DISCUSSION

- Removal modelling:
  - Developed a new model for individuals moving into unobservable states;
  - Matechou et al (2015) has relaxed the assumption of closure within removal models and these methods could be included in the multievent removal framework;
  - Further investigation of the poor performance of near-redundant models.
- Stopover modelling:
  - HMM framework has provided an efficient approach for dealing with complex capture-recapture data;
  - Integrating the analysis of multiple years of data has improved precision and accuracy of parameter estimates;
  - Assessment of goodness-of-fit is an active area of research.

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