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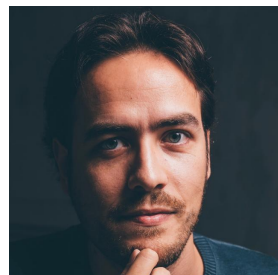
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GUEST EDITORIAL

It is a great honour for me to be a guest editor of The Reasoner, and to introduce you to my conversation with Christian Fermüller. Chris is a logician, working as a professor at the Vienna University of Technology (TU Vienna). A quick glance at [his webpage](#) is sufficient to grasp the great variety of his research interests and outputs, going beyond well-established disciplinary boundaries. As you will find out, overcoming cultural boundaries is also a big part of his non-academic life, with his experiences across different continents. With Chris, we discussed his views on logic, games, philosophy, with a back and



forth between conceptual and technical issues, which I hope will be of interest to the readers of The Reasoner.

PAOLO BALDI
University of Milan

FEATURES

Interview with Christian Fermüller

PAOLO BALDI: Let us start from a fun fact. On your webpage, under the heading “Education” one finds, rather abruptly: “the wrong one”. Can you tell us more about your background?

CHRISTIAN FERMÜLLER : In grammar school (“Gymnasium”), in a very small, provincial town in upper Austria, I got interested in all kinds of subjects, in particular in philosophy, and I decided that I wanted to study philosophy and history of arts. My father, who after all had to finance my studies eventually insisted that I should choose something more “tangible”, obviously worrying about my prospects for making a living. I obliged by enrolling to informatics at TU Vienna, but at the same time I became an eager student of philosophy and history of arts at the venerable University of Vienna. Coming from a rural background, I soon discovered what the big city had to offer: encounters with all kind of people in coffehouses and at parties, discussing philosophy, politics, cinema, literature, music, etc, with a naive seriousness and corresponding excitement that I remember fondly. On the other hand, I regret that I never received a formal education in mathematics.

PB: How did you become interested in logic?

CF: The formal regulations for studying philosophy required to pick a second subject in addition. I initially chose history of arts, but I soon switched to mathematical logic, which at that time, somewhat strangely, was called “Logistik” at the University of Vienna. Obviously, this provided a kind of bridge between informatics and philosophy. Although at that time research and teaching in “Logistik”, was - to put it politely - not (yet) quite up to international standards and to the historical

heritage of Vienna. I was immediately intrigued by the subject. I remember picking up a copy of Hofstadter's "Gödel, Escher, Bach" in Singapore, on the way to Indonesia, which became a kind of second home to me at this time. I also read a few more academic books about logic and recursion theory, but not systematically and without any encouragement by my environment. I spent then an exciting year as a kind of "computer expert" in a provincial place in Indonesia, and after that I returned to Vienna, working as a software engineer with Siemens.

PB: But then you eventually found - and later contributed to shaping - the right environment for research in logic. How did it happen?

CF: At that time I met Alex Leitsch, who had recently become associated professor at TU Vienna, and I got the chance to become an assistant at the computer science department, though initially not in his group. I got to know then also Matthias Baaz, who is not much older than myself, via Alex. Most memorably, Alex arranged for me to join him and Matthias on a visit to Grigory (Grisha) Mints, who at the time, although he was an internationally highly regarded proof theorist, had to share a tiny and shabby office with an indeterminate number of colleagues at a provincial institute in the "Estonian Soviet Socialist Republic". At night I was initiated by Matthias to the mysteries of cut-elimination in the Vodka bar of the very Soviet Viru Hotel in Tallinn, trying not to get too distracted by the many "ladies of the night" surrounding us.

I ended up writing a PhD thesis on resolution methods for the decision problem for fragments of clause logic under Alex's guidance, which also resulted in quite a few joint publications on this and on related topics. This is also a good occasion to mention that in 1995/96 I could spend a whole year at the CSLI in Stanford on a scholarship, by invitation of Grisha Mints, who in the meantime had become a professor there. I really miss his seemingly bottomless supply of knowledge about noteworthy achievements in logic, but also his dry humor. When I asked him what he considered to be the most important question in contemporary logic he promptly replied: "What would Kreisel say?"

PB: Later in your career you got interested in many-valued logic, especially fuzzy logic, which you address mostly using game-based approaches. Can you tell us something about it?

CF: Let me attempt to sketch what interests me in this area by alluding to a concrete example. There is a surprisingly large amount of work on so-called fuzzy quantifiers in the literature. This concept is based, of course, on Zadeh's well known notion of a fuzzy set, i.e., a function from a domain into the real closed unit interval, intended to model a vague predicate, like say, "red". Each object in the domain is taken to be red to some degree in $[0, 1]$; i.e. between, but not excluding, 0 ("not at all") and 1 ("definitely"). A (monadic) fuzzy quantifier, intended to model expressions like "very few", "almost all", "about half", etc, is just a function mapping fuzzy sets (over a fixed domain) into truth values from $[0, 1]$. In this manner one can associate degrees of truth to statements like "Almost all objects of the domain are red". But which truth functions over $[0, 1]$ are ade-

quate and allow us to reason with vague quantifier expressions and predicates in a systematic, robust manner? This is the point where I suggest to employ a game semantic framework. Building on work by Robin Giles from the 1970s, who introduced a game characterizing Łukasiewicz logic, I ask: How can one systematically reduce assertions (formulas) involving a particular fuzzy quantifier to assertions involving involving sub-formulas?

The crucial idea here is that Giles's original proponent-opponent game gets augmented by a random sampling mechanism that is used to model the uncertainty of the players about "correct" classifications. In this manner, truth functions can be extracted from given rules for reducing complex statements. This approach should be contrasted with the usual practice of picking a truth function in a completely ad hoc fashion, not supported by principles of reasoning in an underlying logic.

What interests me here is not so much the technical challenge of proving that certain rules are indeed adequate for certain truth functions, but the attempt to come up with a formal model that can be understood as embedded in a game of giving and asking for reasons (in this case about assertions like "Almost all things here are red") and that can thus be seen as a tool for justifying or at least explicating certain "fuzzy models".

PB: Given your background, it is not surprising that, in parallel to your research on fuzzy logic, you devoted much attention to the related philosophical debate on the analysis of vagueness. On this subject, my next question comes straight from the title of a paper of yours: "What can logicians learn from philosophers?"

CF: The paper you mention was a small and - at least in intention - modest contribution, to make my peers aware of the fact that there are quite different concepts of "reasoning under vagueness" - supervaluationistic, epistemic, pragmatist, contextualist, etc - that each can be associated with different formal semantic structures and corresponding calculi. I presume that what I wanted my peers to "learn" from philosophers was that difficult and subtle non-mathematical questions have to be answered in order to lend any substance to the claim that one deals with a - or even *the* - logic of vagueness when investigating a certain type of many-valued formalism.

PB: Do you think this aim has been achieved? CF: With hindsight I think that even this seemingly modest aim was naive and unrealistic. Of course, there is nothing wrong with just pursuing mathematical interests in the context of many-valued logics. However, I have to admit that I still feel quite embarrassed about colleagues who tell me that they gladly leave "philosophical questions" to others, but who, at the same time, self-confidently claim that, solely by virtue of their technical results about some calculi or algebraic structure, they have something substantial to say about reasoning in presence of vagueness.

To give a concrete example: If you want to offer some version of many-valued logic as a model of reasoning with degrees of truth, one of your commitments that you have to justify is truth functionality. Why should the degree of truth of say "A and B" depend only on the degree of truth of A and the degree of truth of B, and not also on some semantic relation between A and B? The assumption of truth functionality certainly leads to neat and manageable mathematical structures, but it runs counter to robust intuitions about degrees of truth, as e.g. Dorothy Edgington has made clear. Now, this does not necessarily entail that one should forsake all talk of degrees of truth in connection with fuzzy logic. However it does entail an



obligation to reflect on the meaning, possible justifications and implications of truth functionality.

PB: And this isn't "just a philosophical problem", right?

CF: Certainly not! The point is that, also in concrete applications, mathematical neatness is hardly a sufficient reason for adequacy. No one would accept the application of, say, possibility theory, instead of classical probability only because it is formally simpler. In order to succeed, you have to offer robust insights about the informal meaning, implications and underlying assumptions of your mathematical model with respect to alternatives, but in particular also with respect to your intended area of application.

On a more general level, I think that logicians could learn from philosophers about the value of thorough, open-minded, institutionalized debates about their subject matter. Mathematically minded logicians usually shy away from debates about the adequateness and significance of formal concepts, methods and results. They probably think that such discussions just amount to quarrels about the value and importance of their and their colleagues' professional efforts.

PB: I know that you have just received a grant from the Austrian Science Foundation (FWF), for a project entitled "From Semantic Games to Provability Games - and Back". What is the project about?

CF: The title is programmatic: I propose to systematically explore the connections between semantic games, i.e. games for evaluating (possibly graduated) truth of formulas with respect to given interpretations, like Hintikka's game for classical logic, on the one hand, and games that operate on the level of logical consequence and provability, like Lorenzen's game for intuitionistic logic, on the other hand. Of course there already is some work, e.g. of Rahman and Pietarinen on this topic.

My own starting point is a little discovery that I made many years ago and worked out with George Metcalfe: If you start with Giles's semantic game for {Lukasiewicz logic, and if you abstract away from concrete assignments, by considering bundles of possible states for arbitrary, unknown assignments – I call these bundles disjunctive states – then you arrive at a game where the systematic construction of winning strategies corresponds to the logical rules of a so-called hypersequent calculus for {Lukasiewicz logic. I hope to find useful generalizations and variations of this observation.

Related to the just mentioned project, I am pondering a lot these days about interpretations of substructural logics in terms of games about giving and asking for reasons. The idea is to take arguments, i.e. material, rather than logical entailment claims as the basic building blocks of an analysis of logical consequence, that turns out to be thoroughly pluralistic under this point of view. Rather than treating arguments as based on logic, in the sense of truth preservation, I want to treat logic as arising from ways of making explicit how we interact in argumentative, so to speak pre-logical communication. For those familiar with Robert Brandom's pragmatist approach to inferentialism and to logical expressivism, this project should sound appealing and useful.

PB: Are there further research interests that you are pursuing at the moment?

CF: Yes, let me mention a few of them. Together with my Phd student Esther Corsi, I am exploring so-called "logical attack principles", in an attempt to show that particular nonclassical logics emerge from Dung-style argumentation frames.

Such "logical attack principles" for arguments are also the

subject of my recent collaboration with philosopher and psychologist Niki Pfeifer, with whom we devised an empirical study with over 100 students at TU Vienna.

Moreover, with Ondrej Majer we have a long standing plan to write a book on games and many-valued logics, focusing in particular on recovering many-valued logics from equilibria in semantic games under classical (bivalent) evaluations, but imperfect knowledge.

Finally, I want to mention judgment aggregation, a rather recent research field that attracts me, since it still offers room for tackling conceptual problems and exploring alternative logical models. Jointly with friends at the academy of Sciences in Prague – Petr Cintula, Carles Noguera, Marta Bilkova – we are organizing a workshop on this topic in November and I am very much looking forward to interact and, in some cases, meet for the first time top experts like Davide Grossi, Franz Dietrich, Gabriella Pigozzi, and Daniele Porello.

PB: As we have seen, your research interests touch various disciplines: logic, philosophy, computer science and linguistics. What do you think are the particular challenges and pitfalls of interdisciplinary research?

CF: Thank you for this question. The challenge of interdisciplinarity is indeed dear to my heart. I often perceive colleagues to be too quick and superficial in claiming that they care about it. I believe it is fruitful to reserve the expression "interdisciplinary research" to endeavors that do not just involve different subject matters or applications of results of one field to tasks arising in another field. Rather, interdisciplinarity calls for a proper transgression of routines and conventions that are constitutive for a particular field of expertise. It is like learning a new language or getting to know more intimately another culture. But this is actually only the beginning. The real challenge is to achieve results that depend on a genuine interaction of different methodologies and that satisfy criteria of success that apply to different fields. Hence truly interdisciplinary research is also more time consuming and more risky than ordinary research. On the other hand, topics like "models of reasoning" or "argumentation" definitely call for such efforts.

PB: Let us conclude on a light note. I know you have a strong passion for traveling. What are the plans for your next trip?

CF: Very soon I will leave for a trip to Cambodia, which I have visited already more than 20 times since 1991. After at least three decades of travelling, mainly in South-East-Asia, I almost completely stopped to care about sights and "things to do". I try to largely suspend judgment and to get completely immersed into an environment and condition that is radically different from my daily life as a university professor in Vienna.

I can expect to meet there old acquaintances and their children, who now help me to improve my (meager) Khmer language skills, while I help them with their English or math homework. It's a great privilege, – one that is not entirely innocent and unproblematic, of course, given the great asymmetries and travel efforts involved. But we logicians learn to love our problems and paradoxes ...

Formal Ethics, 19–21 June, Ghent

From the 19th to the 21st of June 2019, the 6th edition of *Formal Ethics* was held in Ghent, Belgium. FE2019 was organized by the Centre for Logic and Philosophy of Science and chaired by Federico Faroldi and Frederik Van De Putte, who chose “joint responsibility and collective decision-making” as the special theme for the workshop. Formal ethics is a common denominator for the application of tools from logic, decision theory, game theory, and social choice theory to the analysis of concepts in moral and political philosophy and to the development of ethical theory. The fertility of this relatively young field was demonstrated by the number of extended abstracts that were submitted (over 50), of which the 20 most promising were selected.

The first day was kicked off by keynote speaker Campbell Brown (LSE) who discussed a central concept in population ethics: the valence of a life, and its relation to time and change. As Brown showed, temporal approaches to valence face a certain impossibility result, which he argues can be overcome by domain restriction. The morning continued with talks by Nicolas Cote on liberalism and social choice; Justin Bruner on inequality and the majority rule; and finally, Aleks Knoks on reason-based group choices. After a hearty lunch we listened to Olivier Roy discuss an agent-based model of deliberation and its implications regarding single peakedness and voting cycles; Kai Spiekermann on epistemic injustice as it evolves over informational networks; Satoru Suzuki on measurement-theoretic foundations of weighted utilitarianism; and finally, Tomi Fancsics on impossibility theorems in population axiology.

The second keynote speaker, Edith Elkind (Oxford), presented recent work of herself and co-authors, concerning so-called “approval voting”. In this type of collective decision problems, a set of candidates is selected on the basis of the approvals of each committee member. While seemingly simple, as soon as two or more candidates are to be selected, Elkind showed that interesting trade-offs come about between plausible rationality criteria and computational tractability. The morning session continued with logic talks by Ilaria Canavotto and Alessandro Giordani on causation and accountability in dynamic action logic; Stef Frijters on term modal deontic logics; and Hein Duijf and Allard Tamminga on methodological individualism and logics for group admissibility. In the afternoon, Justin R ger spoke about aggregation and equality and Kacper Kowalczyk presented yet another argument against anti-aggregation. Day two was closed off by Gy rgy Barab s and Andras Szigeti who presented a novel argument for quota-based affirmative action using a simple mathematical model; and Mike Deigan who talked about rational partiality and objective value.

On the last day our third keynote speaker, Ray Briggs (Stanford), talked about some of the problems facing traditional expected utility theory, arguing that in order to amend these we need to adopt a more expansive view of what counts as options in a given decision problem. In particular, on Briggs’ somewhat provocative view, options need not be mutually exclusive, and they needn’t be what “you” (i.e. the agent) can do “now” (i.e. at the point of evaluating decision-theoretic claims). The rest of the morning continued with talks by Jobst Heitzig and

Sarah Hiller on responsibility in extensive-form games with unquantifiable uncertainty; and Abelard Podgorski on what he calls the self-esteem theorem. Our final afternoon session featured Christian Tarsney on structural diversity and metanormative theories; Christopher Meacham on utilitarianism, consent, and the Self-Other asymmetry; and finally Jon Marc Asper on subjective values.

All in all, we can look back at a successful 6th edition of formal ethics which was made possible by the sponsorship of the Flemish Research Foundation (FWO-Vlaanderen), the special Research Fund of Ghent University (BOF UGent), and by the European Research Council (ERC), through the Marie Skłodowska Curie project [DYCODE](#).

THIJS DE CONINCK AND FREDERIK VAN DE PUTTE

Calls for Papers

[IMPRECISE PROBABILITIES, LOGIC AND RATIONALITY](#): special issue of *International Journal of Approximate Reasoning*, deadline 1 October.

[NANCY CARTWRIGHT’S PHILOSOPHY OF SCIENCE](#): special issue of *Theoria*, deadline 1 November.

WHAT’S HOT IN . . .

Medieval Reasoning

Nowadays, when you are traveling around the globe or giving directions to bewildered tourists, English is most likely your go-to *lingua franca*. This is true also in most international academic settings. A lingua franca provides the huge advantage of guaranteeing direct communication, both orally and in writing. Experience teaches that (in some very rare cases) English fails in day-to-day situations, and can do so in some of the most unexpected places. On one memorable occasion, dinner could be had only by communicating with Homeric verses and bits and pieces of Aeolic lyrics. Or that one time when I was organising my very first conference, where one of the attendees could not speak English but offered Polish and Ancient Greek as options. . . Worst case scenario, whenever a common language can’t be found, hand gestures can always save the day (trust me: I am Italian). Nonetheless, it is usually accepted that English is the common language for international academic interactions. (I wish someone had told me that when I was an undergrad, instead of: “everyone should speak their own language and you should be able to understand them all.”) This makes things easier for everyone, but it makes it easiest for native English speakers. One could judge this state of things to be unbalanced, and one might also have strong socio-political opinion about it. But the undisputed predominance of English as the academic lingua franca seems to have had less than stellar consequences on academic philosophy itself: mainstream Anglophone academic philosophy turned out to be both insular and having a disproportionate one-way influence on philosophical traditions in other languages. Eric Schwitzgebel, Linus-Ta Huang, Andrew Higgins and Ivan Gonzales-Cabrera have



an interesting article (fear not: it's in English!) on "The Insularity of Anglophone Philosophy: A Quantitative Analysis" in *Philosophical Papers* (47/2018) [online here](#). Once upon a time, not all the common languages of the past were so unbalanced in favour of one group. Historically, what was originally called *Lingua Franca* (also known as "Sabir") was an odd mix of Northern Italian dialects, Occitan, Catalan, Greek, Berber, and Arabic, along with quite a few additions through time, used primarily for travelling, trading and practical matters, around the Mediterranean Sea and the Western World, roughly from the 11th to the 19th centuries. (If you are curious about this, take a look at Jocelyne Dakhlia (2008): *Lingua Franca – Histoire d'une langue métisse en Méditerranée*). Sabir, of course, was not the first language of common use, neither globally nor in the West. But throughout the Middle Ages (and for quite a while after that!), the *high* lingua franca of the law, of politics and of academia, in the majority of the Western world, was Latin. Medieval Latin was an odd beast at the time, to say the least: neither truly dead nor properly alive, it was a sort of zombie language. And like many a creature that couldn't die properly, not only did it lose a number of features of its previous self (in its syntax, semantics, orthography, phonetics), but it also acquired a pint of blood here and a bite of brain there from its fellow languages that were wandering around alive and well. Most importantly, as every classic horror fan knows, every self-respecting undead entity comes into being by some subversion of the order of nature, be it magical rituals or the occasional mad scientist's experiment: the undead are fundamentally *artificial* things, and so is Medieval Latin – to an obviously higher degree than that of those ordinarily spoken languages which we conventionally call "natural". Truth to be told, Medieval Latin rather resembles a three-headed hydra with dissociative identity issues or a small army of undead siblings at odds with each other. Whereas the language of high communication among the powerful (ML1) could be considered as the baseline, the language of philosophy (ML2) introduces new uses, special meanings, restrictions – it is, in other words, more regimented than ML1. This tendency of regimentation is even more evident in the language of medieval logic (ML3), where the ordinary semantics and grammatical uses are overthrown more than once. ML3 is an extremely regimented language, with a high degree of precision and expressive power. However, ML3 is what most Humanist and Renaissance stylists would call *ugly* – not that ML2 and ML1 are much better. Ugly as the prose of all medieval academic Latin may be, it is technical enough to properly treat the subjects it was supposed to treat and articulate the arguments it is supposed to articulate. While most medieval philosophers were not witty essayists like some of their later colleagues, they were on equal footing among their peers: nobody was a Latin native speaker and everyone went roughly through the same *cursum studuorum* to learn the basics.

To be continued. . .

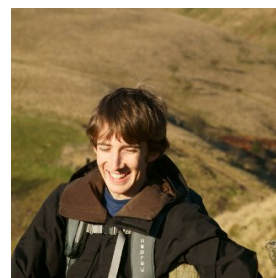
GRAZIANA CIOLA
Durham University

Uncertain Reasoning

I thought I'd use my column this month to mention a few free resources for learning uncertain reasoning. This topic for the column was prompted by the recent publication of [The Open Handbook of Formal Epistemology](#) which contains eleven chapters on various topics in formal epistemology writ-

ten by experts in the field. I haven't read all the chapters yet, but those I have read are uniformly great. Readers interested in getting a firm foundation in what's going on in formal epistemology at the moment would do well to just read this cover to cover.

I expect that part of the inspiration for the Formal Epistemology handbook was [the Open Logic Project](#). The original open access textbook on which the OLP books were originally based was P.D. Magnus' "forallx" which has been around since 2005. The Open Logic Project now consists of one huge book that covers a great deal of logic, or several more manageably sized books which cover specific topics from the complete text. It's a great project, and among the various versions of the textbook is one to suit most sorts of logic course. Of course, most of this material is more about reasoning under certainty than it is about uncertain reasoning, but I think it's still worth mentioning in this context.



Another excellent resource is [the list of other free and open logic texts](#) maintained by the Open Logic Project. Included on that list, towards the bottom are some texts on formal epistemology, I can personally recommend the Weatherson notes, which I have used to teach, and the Schwarz text, which I had a brief look at.

Then we have classic works that are now out of copyright and thus freely available, like Boole's "Laws of Thought" or Keynes' "Theory of Probability". These are probably of more interest as historical documents rather than good options to learn about uncertain reasoning in the modern day.

Finally, I would like to mention another resource for open textbooks, [the open textbook library](#). This has a surprisingly rich collection of (mostly introductory) textbooks on a broad range of topics. There are textbooks of interest to uncertain reasoners spread across a number of sections of this site, including Computer Science, Economics, Philosophy, Mathematics, and the Natural and Social Sciences. This looks like a great resource that I have only just begun to explore properly.

And now, a short digression directed mainly at those of you who teach. Textbooks can be very expensive, and so, if adequate open texts are available, you should really consider using them in your teaching. Anything we can do to ease the financial burden on our students must be a good thing. Texts available in a variety of electronic formats can also help with issues of accessibility (increasing the font size for students with restricted vision or text-to-speech software for hard of hearing students, for example).

Many of these resources, as well as being "free as in beer" – available for no money – are "free as in speech" – available openly, for you to use as you see fit. ([See here](#) for more on the distinction, albeit in a software context,). One advantage of genuinely open textbooks is their flexibility: that permission to remix or adapt a text to your needs is incredibly powerful, and something that many people can take advantage of (as the wealth of versions of the Open Logic Project text makes clear). Many of the same arguments made in favour of "free software"

hold in this case too, for example, the articles listed [here](#).

SEAMUS BRADLEY

Philosophy, University of Leeds

Mathematical Philosophy

Whenever we encounter a proof of a mathematical theorem—whether in a textbook, on a blackboard, or just in our heads—we are guaranteed that its premises imply its conclusion. Of course, the proof may rely on axioms that are only given implicitly, but it should always be possible to make those assumptions explicit. Often, however, not all of those premises are genuinely *necessary* in order to derive the conclusion. So the question arises: given some mathematical theorem, which axioms are strictly required to prove it? For example, can the Axiom of Choice be eliminated from proofs of the Hahn-Banach theorem?

Reverse mathematics is a branch of mathematical logic that addresses this question. While the eliminability of some principle can be shown by providing an alternative proof that does not rely on that principle, non-eliminability can often be harder to establish. Reverse mathematics provides a way to do so: let's see how it works.

The first step is to formalise the theorem. Reverse mathematics does this in the language of *second-order arithmetic*, which consists of the language of first-order arithmetic augmented with quantifiers that range over sets of natural numbers, and variables that denote those sets. Sometimes, this process involves complex coding, most notably when we wish to talk about uncountable objects and structures like continuous functions or metric spaces, since the language of second-order arithmetic can only directly represent countable objects. Once the theorem is expressed in this language, we show that it follows from the axioms of some subsystem of second-order arithmetic S , whose axioms state that sets of a certain complexity exist. One then “reverses” the process by adding the theorem to a weak base theory called RCA_0 (a theory that states that computable sets exist, where the ‘0’ subscript indicates that we are working with restricted induction), and deriving the axioms of S . This “reversal” shows that the axioms of S are strictly necessary in order to prove the theorem.

Reverse mathematics was established in the mid-1970s by Harvey Friedman, and it is now a flourishing field of research with many interesting results. The most striking of these results is perhaps the emergence of a pattern by which most theorems of ordinary mathematics cluster around five natural subsystems of second-order arithmetic, also known as the “Big Five”. This phenomenon is comprehensively explored in Stephen Simpson’s seminal textbook *Subsystems of Second Order Arithmetic* (Cambridge University Press, 1999). The Big Five subsystems are often taken to formally capture specific views on the foundations of mathematics: RCA_0 for finitism, WKL_0 (RCA_0 extended by the addition of Weak König’s Lemma) for finitistically reducible systems, ACA_0 (RCA_0 plus arithmetical comprehension) for predicativism, ATR_0 (RCA_0 plus arithmetical trans-

finite recursion) for predicatively reducible systems, $\Pi_1^1\text{-CA}_0$ (RCA_0 plus Π_1^1 comprehension) for impredicative arithmetic. These reversals thus classify both theorems and axioms of subsystems of second-order arithmetic in equivalence classes. In this way, many mathematical theorems are linearly ordered in a nice hierarchy by their proof-theoretic strength. In recent years, however, this neat picture has been called into question by the discovery of theorems which lie outside the Big Five and are now studied as part of the “Reverse Mathematics Zoo”. Noteworthy examples include combinatorial theorems such as Ramsey’s theorem for pairs; the interested reader can consult Denis Hirschfeldt’s excellent book *Slicing the Truth* (World Scientific, 2014).

When I first came into contact with reverse mathematics, it struck me as a field full of exciting philosophical questions: To what extent is the formalised version of a mathematical theorem faithful to the meaning of the informal statement? What is the significance of reversals from mathematical theorems to set existence principles? How can we understand the conceptual dependencies between axioms and theorems in the light of reversals? What common aspects of mathematical theorems are uncovered when two otherwise unrelated mathematical theorems are shown to reverse to the same set existence principle? Can tools from reverse mathematics be fruitfully applied to philosophical questions? For this reason, reverse mathematics is an important part of my current project entitled “[Formalism, Formalization, Intuition and Understanding in Mathematics](#)”, in partnership with the Archives Henri-Poincaré at the Université de Lorraine and the Institut d’Histoire et de Philosophie des Sciences et des Techniques (IHPST) in Paris, which investigates how the formalisation of different areas of informal mathematics helps to sharpen our understanding of those areas.

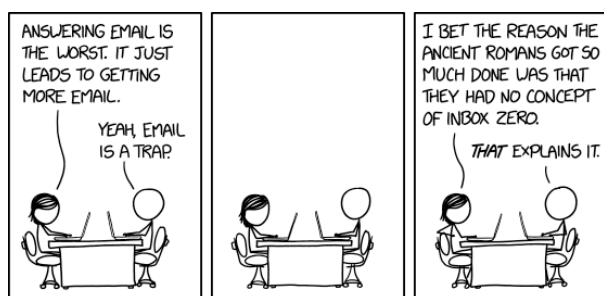
However, the philosophy of reverse mathematics is still in its infancy. Historically informed presentations of the field can be found in a great paper by Walter Dean and Sean Walsh (“[The Prehistory of the Subsystems Of Second-Order Arithmetic](#)”, *The Review of Symbolic Logic*, 2017) and in a book by the historian of mathematics John Stillwell (*Reverse Mathematics: Proofs from the Inside Out*, Princeton University Press, 2018). Philosophical issues such as the nature of set existence principles are addressed by Benedict Eastaugh (MCMP) in his paper “[Set Existence Principles and Closure Conditions](#)” (*Philosophia Mathematica*, 2019) and in his forthcoming Stanford Encyclopedia of Philosophy entry. In a joint paper in progress, Benedict and I suggest that the very notion of “ordinary mathematics” used in the characterisation of the scope and significance of reverse mathematics may not be as stable a notion as we might initially think.

There is still much work to do in order to achieve a good grasp of reverse mathematics and its philosophical significance and ramifications. To foster explicit discussion on the history and philosophy of reverse mathematics, I applied for additional funding from the [French-Bavarian Cooperations](#) together with Andrew Arana (IHPST, Paris) to organise two four-day seminars in Paris and Munich on the history and philosophy of reverse mathematics. Four researchers and graduate students from each institution participated, together with Walter Dean (Warwick University) and Ryota Akiyoshi (Keio University), to discuss questions such as: How compelling are the correspondences between foundational approaches and subsystems of second-order arithmetic? How did reverse mathematics emerge from the informal study of mathematical equivalences?



What does the Reverse Mathematics Zoo tell us about the significance of the reverse mathematics hierarchy? The discussion was very stimulating and productive, and some individual and joint papers originating from the discussion are already under way. In the light of its intrinsic philosophical interest, we can expect that reverse mathematics and its philosophy will occupy an increasingly prominent role in mathematical philosophy. Stay tuned!

MARIANNA ANTONUTTI
Munich Centre for Mathematical Philosophy



EVENTS

AUGUST

SEH: International Conference on Science, Education, and Humanities, Perak, Malaysia, 16 August.

SEPTEMBER

EoAS: Epistemology of Analogue Simulation, University of Geneva, 10 September.

WAP: Workshop: Assertion and Proof, Lecce, Italy, 12–14 September.

OWG?: On What Grounds? Grounding, Dependence, and Fundamentality, University of Geneva, Switzerland, 17 September.

SMfM-A Statistical Methods for Meta-Analysis, University of Southampton 17–18 September

OCTOBER

BIRDS: Bridging the Gap between Information Science, Information Retrieval and Data Science, Melbourne, Australia, 19 October.

COURSES AND PROGRAMMES

Courses

SSA: Summer School on Argumentation: Computational and Linguistic Perspectives on Argumentation, Warsaw, Poland, 6–10 September.

Programmes

APHIL: MA/PhD in Analytic Philosophy, University of Barcelona.

MASTER PROGRAMME: MA in Pure and Applied Logic, University of Barcelona.

DOCTORAL PROGRAMME IN PHILOSOPHY: Language, Mind and Practice, Department of Philosophy, University of Zurich, Switzerland.

DOCTORAL PROGRAMME IN PHILOSOPHY: Department of Philosophy, University of Milan, Italy.

LOGICS: Joint doctoral program on Logical Methods in Computer Science, TU Wien, TU Graz, and JKU Linz, Austria.

HPSM: MA in the History and Philosophy of Science and Medicine, Durham University.

MASTER PROGRAMME: in Statistics, University College Dublin.

LoPhiSC: Master in Logic, Philosophy of Science and Epistemology, Pantheon-Sorbonne University (Paris 1) and Paris-Sorbonne University (Paris 4).

MASTER PROGRAMME: in Artificial Intelligence, Radboud University Nijmegen, the Netherlands.

MASTER PROGRAMME: Philosophy and Economics, Institute of Philosophy, University of Bayreuth.

MA IN COGNITIVE SCIENCE: School of Politics, International Studies and Philosophy, Queen's University Belfast.

MA IN LOGIC AND THE PHILOSOPHY OF MATHEMATICS: Department of Philosophy, University of Bristol.

MA PROGRAMMES: in Philosophy of Science, University of Leeds.

MA IN LOGIC AND PHILOSOPHY OF SCIENCE: Faculty of Philosophy, Philosophy of Science and Study of Religion, LMU Munich.

MA IN LOGIC AND THEORY OF SCIENCE: Department of Logic of the Eotvos Lorand University, Budapest, Hungary.

MA IN METAPHYSICS, LANGUAGE, AND MIND: Department of Philosophy, University of Liverpool.

MA IN MIND, BRAIN AND LEARNING: Westminster Institute of Education, Oxford Brookes University.

MA IN PHILOSOPHY: by research, Tilburg University.

MA IN PHILOSOPHY, SCIENCE AND SOCIETY: TiLPS, Tilburg University.

MA IN PHILOSOPHY OF BIOLOGICAL AND COGNITIVE SCIENCES: Department of Philosophy, University of Bristol.

MA IN RHETORIC: School of Journalism, Media and Communication, University of Central Lancashire.

MA PROGRAMMES: in Philosophy of Language and Linguistics, and Philosophy of Mind and Psychology, University of Birmingham.

MRES IN METHODS AND PRACTICES OF PHILOSOPHICAL RESEARCH: Northern Institute of Philosophy, University of Aberdeen.

MSc IN APPLIED STATISTICS: Department of Economics, Mathematics and Statistics, Birkbeck, University of London.

MSc IN APPLIED STATISTICS AND DATAMINING: School of Mathematics and Statistics, University of St Andrews.

MSc IN ARTIFICIAL INTELLIGENCE: Faculty of Engineering, University of Leeds.

MSc IN COGNITIVE & DECISION SCIENCES: Psychology, University College London.

MSc IN COGNITIVE SYSTEMS: Language, Learning, and Reasoning, University of Potsdam.

MSc IN COGNITIVE SCIENCE: University of Osnabrück, Germany.

MSc IN COGNITIVE PSYCHOLOGY/NEUROPSYCHOLOGY: School of Psychology, University of Kent.

MSc IN LOGIC: Institute for Logic, Language and Computation, University of Amsterdam.

MSc IN MIND, LANGUAGE & EMBODIED COGNITION: School of Philosophy, Psychology and Language Sciences, University of Edinburgh.

MSc IN PHILOSOPHY OF SCIENCE, TECHNOLOGY AND SOCIETY: University of Twente, The Netherlands.

MRES IN COGNITIVE SCIENCE AND HUMANITIES: LANGUAGE, COMMUNICATION AND ORGANIZATION: Institute for Logic, Cognition, Language, and Information, University of the Basque Country (Donostia San Sebastián).

OPEN MIND: International School of Advanced Studies in Cognitive Sciences, University of Bucharest.

RESEARCH MASTER IN PHILOSOPHY AND ECONOMICS: Erasmus University Rotterdam, The Netherlands.

JOBS AND STUDENTSHIPS

Jobs

POSTDOCTORAL RESEARCHER: in Logic on the project Logical Foundations and Applications of Depth-Bounded Probability, University of Milan, deadline 6 September.

LECTURER: in Theoretical Philosophy, University of the Witwatersrand, Johannesburg, deadline 31 August.

RESEARCH FELLOW: in Philosophy of Science, University of Oslo, deadline 31 August.

BERTRAND RUSSELL PROFESSORSHIP OF PHILOSOPHY: in Theoretical Philosophy, University of Cambridge, deadline 18 October.

Studentships

PhD position in Research Methods in Science and Technology, University of Urbino deadline 27 August

