Adverse Selection, Loss Coverage and Equilibrium Premium in Insurance Markets

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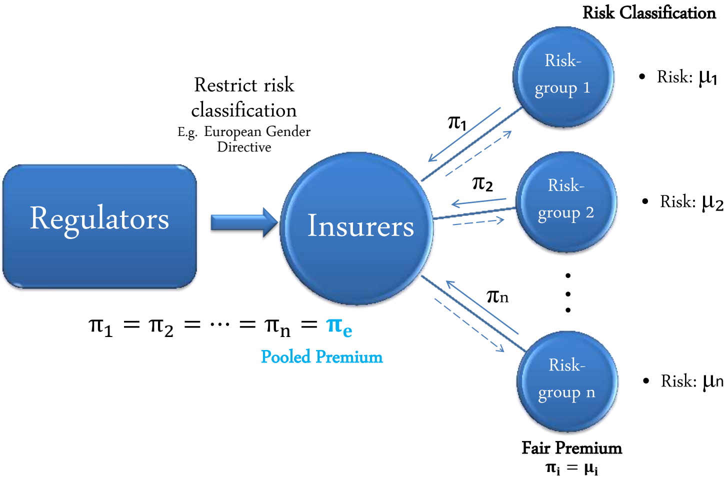
Background

How insurance works and risk classification scheme



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Adverse Selection

- $0, \pi_1, \pi_2, \pi_3, \pi_e, \ldots, \pi_7, \pi_8, \ldots, \pi_n, 1.$

Adverse Selection

- $0, \pi_1, \pi_2, \pi_3, \pi_e, ..., \pi_7, \pi_8, ..., \pi_n, 1.$

Typical definition

Purchasing decision is positively correlated with losses
-Chiappori and Salanie (2000) “Positive Correlation Test”

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- Empirical results are mixed and vary by market.

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- Empirical results are mixed and vary by market.

Life Insurance	Cawley and Philipson (1999)	X
Auto Insurance	Chiappori and Salanie (2000)	X
	Cohen (2005)	O
Annuity	Finkelstein and Poterba (2004)	O
Health Insurance	Cardon and Hendel (2001)	X

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Definition

$$\text{Adverse Selection (AS)} = \frac{\text{expected claim per policy}}{\text{expected loss per risk}} = \frac{E[QL]}{E[Q]E[L]}, \quad (1)$$

where Q: quantity of insurance; L: risk experience.

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$$\text{Adverse Selection Ratio: } S \quad = \quad \frac{\text{AS at pooled premium } \pi_e}{\text{AS at risk-differentiated premiums}} \tag{2}$$

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$$\begin{aligned}\text{Adverse Selection Ratio: } S &= \frac{\text{AS at pooled premium } \pi_e}{\text{AS at risk-differentiated premiums}} \\ &> 1 \Rightarrow \textbf{Adverse Selection}.\end{aligned} \tag{2}$$

Example

Example

- A population of 1000
- Two risk groups
  - 200 high risks with risk 0.04
  - 800 low risks with risk 0.01
- No moral hazard

Example

Full risk classification

Example

Full risk classification

	Low risks	High risks	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums (**differentiated**)	0.01	0.04	0.016
Numbers insured	400	100	500
Adverse Selection Ratio (S)			1

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Full risk classification

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Restriction on risk classification-Case 1

Example

Restriction on risk classification-Case 1

	Low risks	High risks	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums (**pooled**)	0.02	0.02	0.02
Numbers insured	300(400)	150(100)	450(500)
Adverse Selection Ratio (S)			1.25>1

Example

Restriction on risk classification-Case 1

	Low risks	High risks	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums (**pooled**)	0.02	0.02	0.02
Numbers insured	300(400)	150(100)	450(500)
Adverse Selection Ratio (S)			1.25>1

Moderate adverse selection

Example

Restriction on risk classification-Case 2

Example

Restriction on risk classification-Case 2

	Low risks	High risks	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums (**pooled**)	0.02154	0.02154	0.02154
Numbers insured	200(400)	125(100)	325(500)
Adverse Selection Ratio (S)			1.3462>1

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Heavier adverse selection

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Heavier adverse selection

Adverse selection suggests pooling is always bad. But is it?

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Loss Coverage

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$$\text{Loss Coverage (LC)} = \frac{\text{insured expected losses}}{\text{population expected losses}} \tag{3}$$

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- Aim of insurance: provide protection for those who suffer losses.
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- Thomas (2008, 2009) "**Loss Coverage**":

Definition

$$\text{Loss Coverage (LC)} = \frac{\text{insured expected losses}}{\text{population expected losses}} \quad (3)$$

$$\text{Loss Coverage Ratio: } C = \frac{\text{LC at a pooled premium } \pi_e}{\text{LC at at risk-differentiated premium } \pi_i} \quad (4)$$

$$> 1, \textbf{Favorable!}$$

Example

No restriction on risk classification

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No restriction on risk classification

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Total population	800	200	1000
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Numbers insured	400	100	500
Insured losses	4	4	8
Loss coverage ratio (C)			1

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Numbers insured	400	100	500
Insured losses	4	4	8
Loss coverage ratio (C)			1

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Insured losses	3	6	9
Loss coverage ratio (C)			1.125>1

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Insured losses	3	6	9
Loss coverage ratio (C)			1.125>1

Moderate adverse selection ($S = 1.25$) but favorable loss coverage.

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Heavier adverse selection ($S = 1.3462$) and worse loss coverage.
Loss coverage might be a better measure!

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Definition

Demand elasticity: $\epsilon(\mu, \pi) = -\frac{\partial d(\mu,\pi)}{d(\mu,\pi)} / \frac{\partial \pi}{\pi}$ i.e. sensitivity of demand to premium changes.

Demand Function

Iso-elastic demand function

$$\epsilon(\mu, \pi) = \lambda, \text{ i.e. constant} \tag{5}$$

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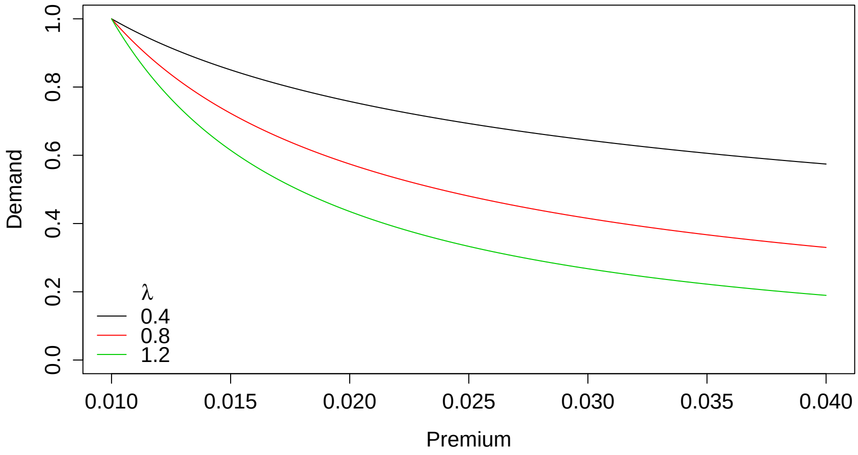
Iso-elastic demand function

$$\epsilon(\mu, \pi) = \lambda, \text{ i.e. constant} \tag{5}$$

$$d(\mu, \pi) = \tau \left[\frac{\pi}{\mu}\right]^{-\lambda}. \tag{6}$$

Iso-elastic demand function

$\tau = 1, \mu = 0.01, \lambda = 0.4, 0.8$ and 1.2



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If $\lambda_1 = \lambda_2 = \lambda$,

$$\pi_e = \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1^{\lambda} + \alpha_2 \mu_2^{\lambda}}, \qquad (8)$$

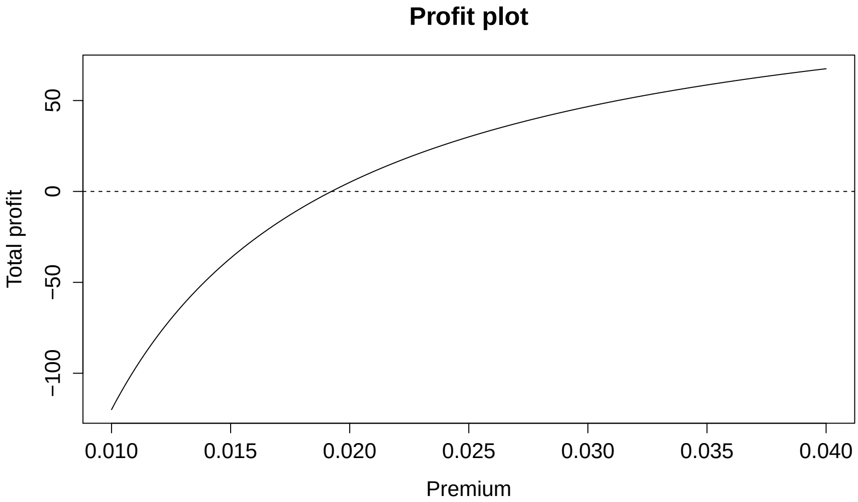
where

$$\alpha_i = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}, i = 1, 2 \qquad (9)$$

(Fair-premium demand-share)

Unique equilibrium premium

$p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; p_2 = 1000, \tau_2 = 1, \mu_2 = 0.04, \lambda_1 = \lambda_2 = 1$



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Results on adverse selection

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Adverse Selection Ratio

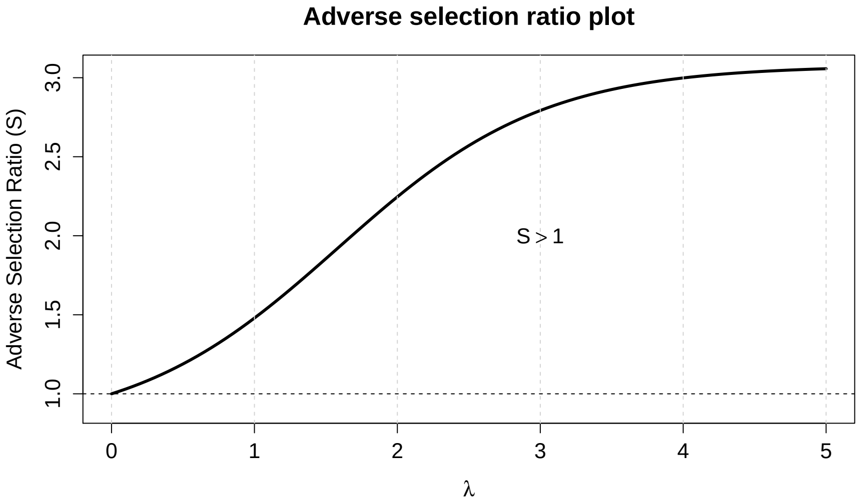
$$S = \frac{\pi_e}{\alpha_1\mu_1 + \alpha_2\mu_2}. \tag{10}$$

$$\alpha_i = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}, i = 1, 2$$

(Fair-premium demand-share)

Results: Adverse Selection Ratio (S)

$p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; p_2 = 1000, \tau_2 = 1, \mu_2 = 0.04$



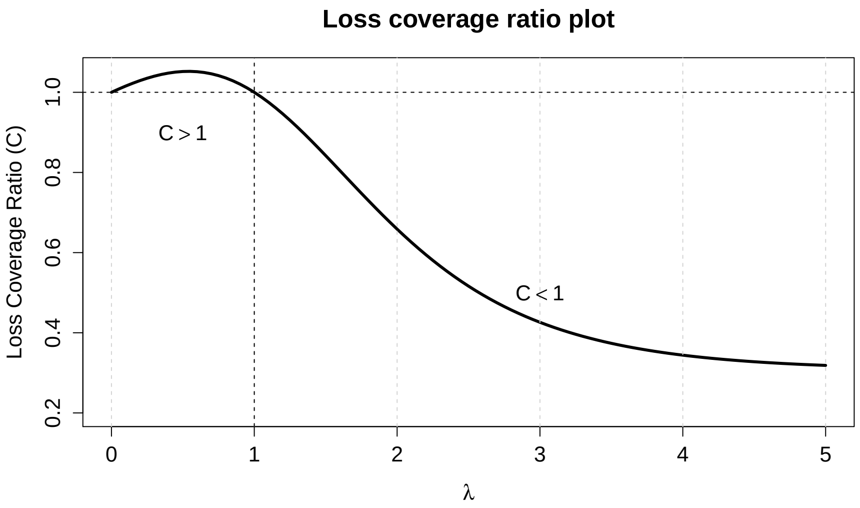
Results on loss coverage

Loss Coverage Ratio

$$C = \frac{1}{\pi_e{}^{\lambda}} \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1 + \alpha_2 \mu_2}. \tag{11}$$

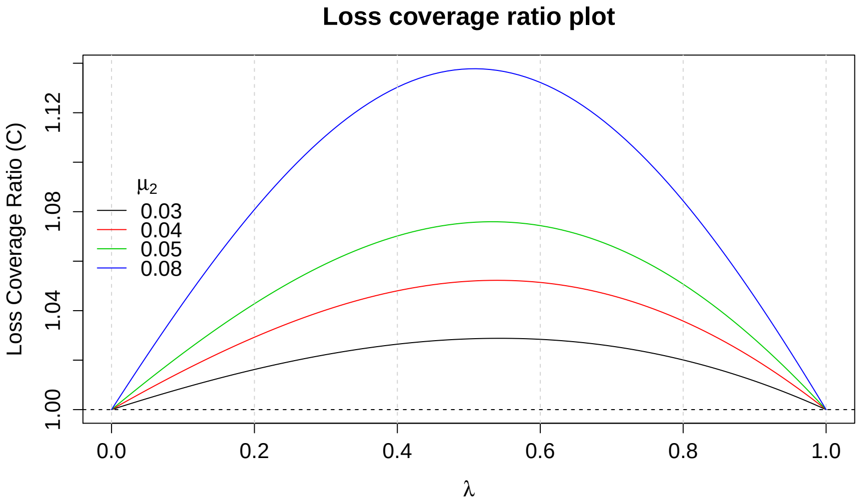
Results: Loss Coverage Ratio (C)

$p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; p_2 = 1000, \tau_2 = 1, \mu_2 = 0.04$



Results: Loss Coverage Ratio (C)

$p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; p_2 = 1000, \tau_2 = 1, \mu_2 = 0.03, 0.04, 0.05, 0.08$



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- When there is restriction on risk classification, a pooled premium π_e is charged across all risk-groups.
- There will always be adverse selection $\Rightarrow$ Adverse selection may not be a good measure.
- Loss coverage is an alternative metric.
- **Adverse selection is not always a bad thing!**
  A moderate level of adverse selection can increase loss coverage.

Further Research

- Other/more general demand e.g. $d(\mu, \pi) = \tau e^{1-(\frac{\pi}{\mu})^{\lambda}}$.
- Loose restriction on demand elasticities.
- Partial restriction on risk classification.

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Questions?

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Thank you!