Adverse Selection, Loss Coverage and Equilibrium Premium in Insurance Markets

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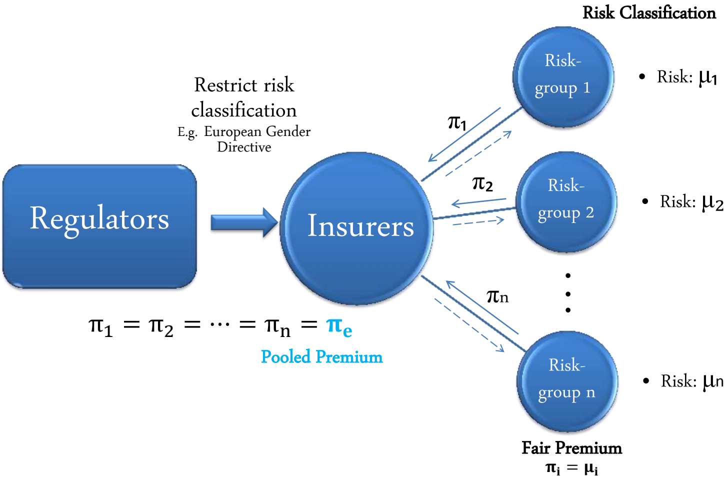
Background

How insurance works and risk classification scheme



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Adverse Selection

- $0, \pi_1, \pi_2, \pi_3, \pi_e, \ldots, \pi_7, \pi_8, \ldots, \pi_n, 1.$

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Original definition

Purchasing decision is positively correlated with losses
-Chiappori and Salanie (2000) “Positive Correlation Test”

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- Empirical results are mixed and vary by market.

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- Empirical results are mixed and vary by market.

Life Insurance	Cawley and Philipson (1999)	X
Auto Insurance	Chiappori and Salanie (2000)	X
	Cohen (2005)	O
Annuity	Finkelstein and Poterba (2004)	O
Health Insurance	Cardon and Hendel (2001)	X

Adverse Selection

- Restricting risk classification $\Rightarrow$ Policy is over-subscribed by high risks **BAD?**

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- Model:

$$S = \frac{E[QL]}{E[Q]E[L]} = \frac{\text{pooled premium } \pi_e}{\text{population-weighted fair premium}} \tag{1}$$

where

$$\begin{aligned} Q &: \text{ quantity of insurance} \\ L &: \text{ risk experience} . \end{aligned}$$

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- **S > 1 $\Rightarrow$ Adverse Selection.**

Example

Example

- A population of 1000
- Two risk groups
  - 200 high risks with risk 0.04
  - 800 low risks with risk 0.01
- No moral hazard

Example

No restriction on risk classification

Example

No restriction on risk classification

Table 1	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium (**fair premium**)	0.01	0.04
Number insured	400	100
Adverse Selection	1	

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Restriction on risk classification-Case 1

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Table 2	Low risk-group	High risk-group
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Break-even premium (**pooled premium** π_e)	0.02	
Number insured	300(400)	150(100)
Adverse Selection	1.25>1	

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Moderate adverse selection

Example

Restriction on risk classification-Case 2

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Table 3	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium (**pooled premium** π_e)	0.02154	
Number insured	200(400)	125(100)
Adverse Selection	**1.3462>1**	

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Table 3	Low risk-group	High risk-group
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Heavier adverse selection

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Adverse selection suggests pooling is always bad. But is it?

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Loss Coverage

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Definition

$$\begin{aligned} \text{Loss Coverage} &= \frac{\text{insured expected losses}}{\text{population expected losses}} \\ \text{Loss Coverage Ratio} &= \frac{\text{loss coverage at a pooled premium}\pi_e}{\text{loss coverage at at fair premium}\pi_i} \\ &> 1, \textbf{Favorable!} \end{aligned}$$

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Moderate adverse selection but favorable loss coverage.

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Insured expected losses	2	5
Loss Coverage	0.4375	
Loss Coverage Ratio	**0.875<1**	

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Heavier adverse selection and worse loss coverage.
Loss Coverage might be a better measurement!

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Demand Functions

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Definition

The demand elasticity $\epsilon(\mu, \pi) = -\frac{\partial d(\mu,\pi)}{d(\mu,\pi)} / \frac{\partial \pi}{\pi}$ i.e. sensitivity of demand to premium changes.

Demand Functions

Iso-elastic demand

$$\begin{aligned} d(\mu,\pi) &= \tau\left[\frac{\pi}{\mu}\right]^{-\lambda} \\ \epsilon(\mu,\pi) &= \lambda \end{aligned}$$

Negative-exponential demand

$$\begin{aligned} d(\mu,\pi) &= \tau e^{(1-\frac{\pi}{\mu})\lambda} \\ \epsilon(\mu,\pi) &= \frac{\lambda}{\mu}\pi \end{aligned}$$

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Equilibrium Premium

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$$f(\pi_e) = d(\mu_1, \pi_e)p_1(\pi_e - \mu_1) + d(\mu_2, \pi_e)p_2(\pi_e - \mu_2) = 0. \qquad (2)$$

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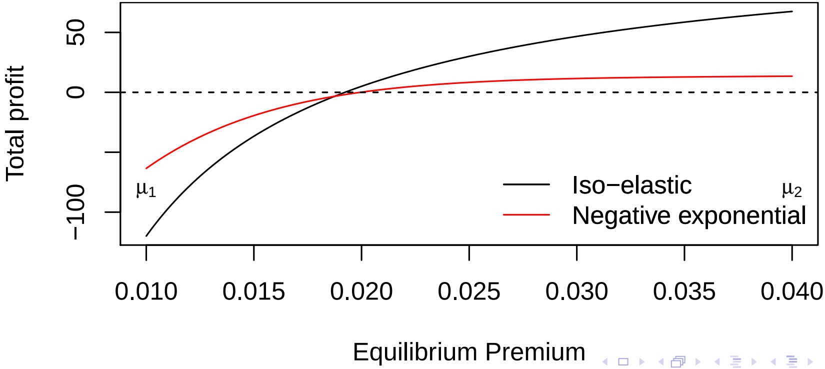
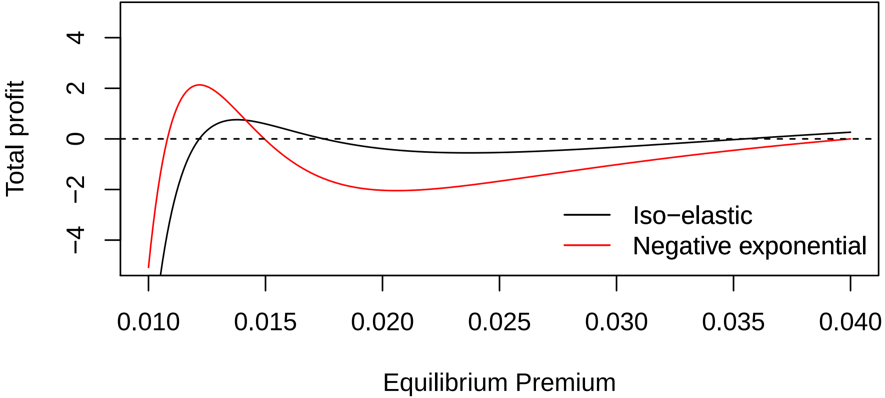


Multiple Equilibria

Only for extreme parameter values. E.g.
$p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01, \lambda_1 = 5; p_2 = 80, \tau_2 = 1, \mu_2 = 0.04, \lambda_2 = 1$



Multiple Equilibria

Theorem

Given $(\mu_1, \mu_2), (\tau_1, \tau_2)$ *and* (λ_1, λ_2)*, there are multiple equilibria if and only if* $c < c_1$ *and* $\alpha(\pi_{01}) \leq \alpha \leq \alpha(\pi_{02})$*.*
Where

- $\alpha = \frac{p_1}{p_2}$.
- π_{01}, π_{02} *are solutions to* $f(\pi_e) = 0, f'(\pi_e) \leq 0$.

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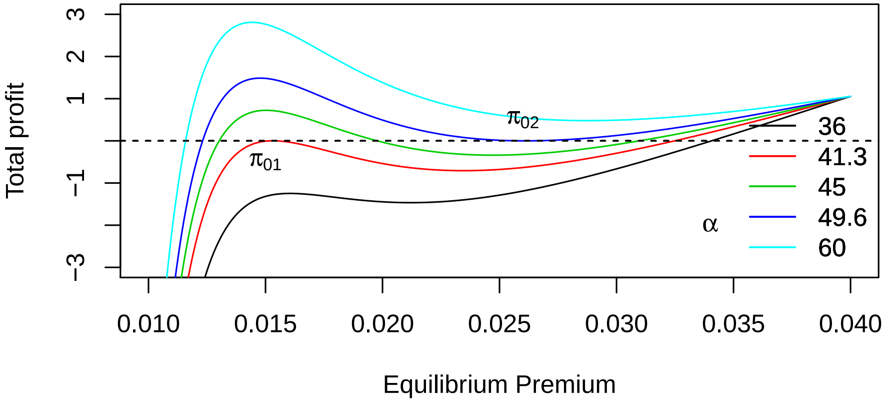
Where

- $\alpha = \frac{p_1}{p_2}$.
- π_{01}, π_{02} *are solutions to* $f(\pi_e) = 0, f'(\pi_e) \leq 0$.

For iso-elastic demand, $c = \lambda_2 - \lambda_1, c_1 = -\frac{\sqrt{\mu_1}+\sqrt{\mu_2}}{\sqrt{\mu_2}-\sqrt{\mu_1}} < 0$.
For negative-exponential demand, $c = \frac{\lambda_2}{\mu_2} - \frac{\lambda_1}{\mu_1}, c_1 = -\frac{4}{\mu_2-\mu_1} < 0$.

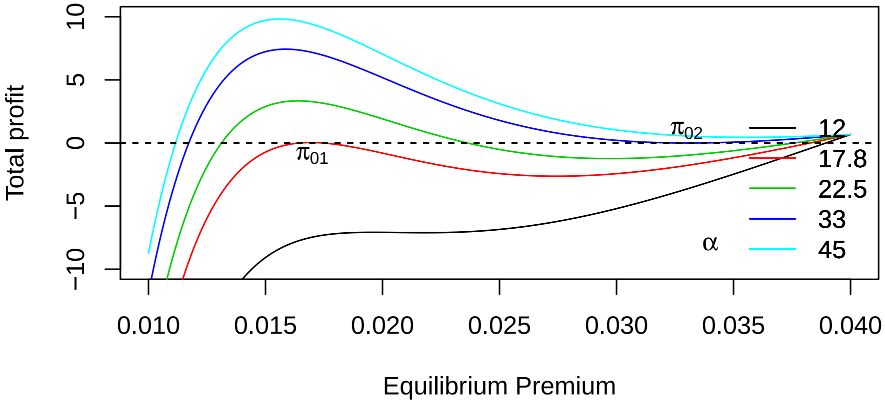
Example: Iso-elastic demand

$\mu_1 = 0.01, \mu_2 = 0.04 \Rightarrow c_1 = -3;$
$\lambda_1 = 4, \lambda_2 = 0.5 \Rightarrow c = -3.5 < c_1$



Example: Negative-exponential demand

$\mu_1 = 0.01, \mu_2 = 0.04 \Rightarrow c_1 = -133.33$:
$\lambda_1 = 2, \lambda_2 = 0.5 \Rightarrow c = -187.5 < c_1$



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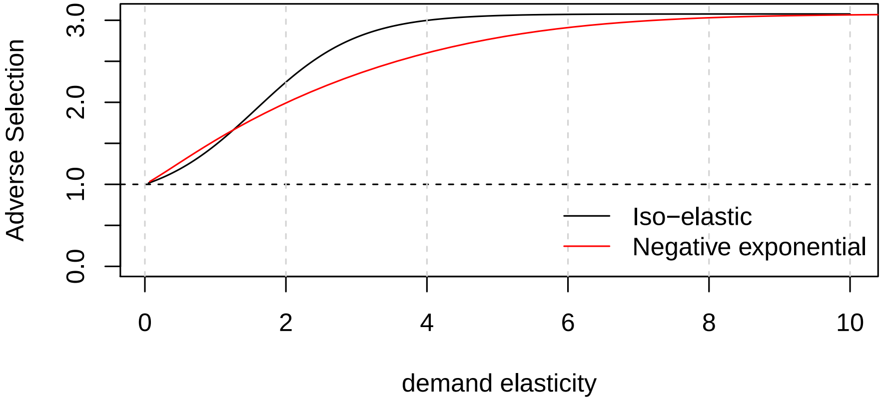
Results

Assumptions

- There are 2 risk-groups
- They have equal demand elasticities $\Rightarrow$ **Unique Equilibrium**
  - Iso-elastic demand: $\lambda_1 = \lambda_2 = \epsilon(\pi_e)$
  - Negative-exponential demand: $\frac{\lambda_1}{\mu_2}\pi_e = \frac{\lambda_2}{\mu_2}\pi_e = \epsilon(\pi_e)$

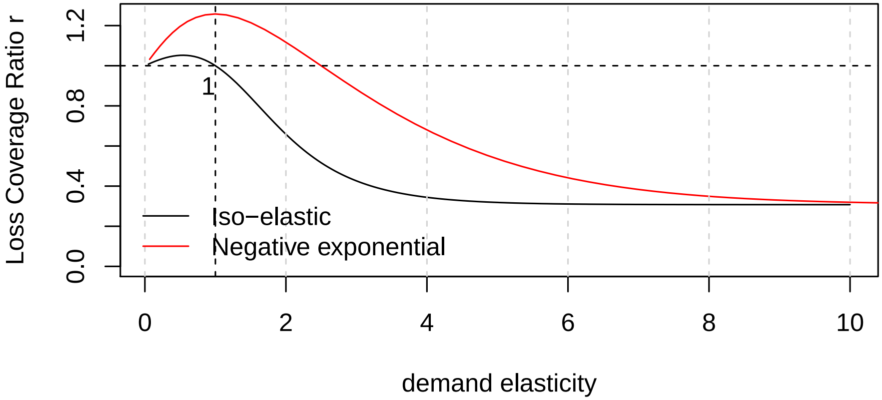
Results: Adverse Selection

$p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; p_2 = 1000, \tau_2 = 1, \mu_2 = 0.04$



Results: Loss Coverage

$p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; p_2 = 1000, \tau_2 = 1, \mu_2 = 0.04$



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Summary

- When there is restriction on risk classification, a pooled premium π_e is charged across all risk-groups.
- There will always be adverse selection $\Rightarrow$ Adverse Selection may not be a good measurement.
- Loss Coverage is an alternative metric.
  Using iso-elastic and negative-exponential demand,
- **Adverse Selection is not always a bad thing!**
  A moderate level of adverse selection can increase loss coverage.

Further Research

- Other/more general demand e.g. $d(\mu, \pi) = \tau e^{1-(\frac{\pi}{\mu})^{\lambda}}$.
- Loose restriction on demand elasticities.
- Partial restriction on risk classification.

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Questions?

Thank you!