Multiple Equilibria, Adverse Selection and Loss Coverage in Insurance Markets

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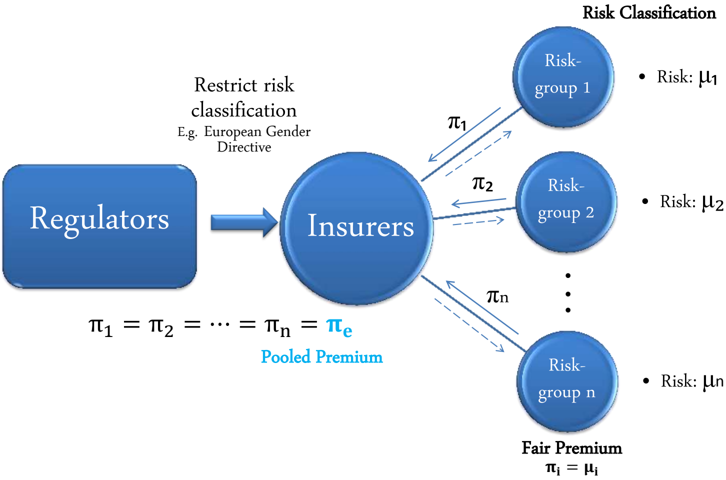
Background

How insurance works and risk classification scheme



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Demand Functions

Definition

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- $0 < d(\mu, \pi) < 1$.
- $\frac{\partial}{\partial \pi} d(\mu, \pi) < 0$, which implies that demand falls as the premium rises.
- $\frac{\partial^2}{\partial \pi^2} d(\mu, \pi) > 0$, which implies a decreasing rate of fall in demand as premium increases. I.e. individuals are risk averse.

Demand Functions

Iso-elastic demand

$$\begin{aligned} d(\mu,\pi) &= \tau\left[\frac{\pi}{\mu}\right]^{-\lambda} \\ \epsilon(\mu,\pi) &= -\frac{\pi}{d(\pi,\mu)}\frac{\partial}{\partial\pi}d(\mu,\pi) \\ &= \lambda \end{aligned}$$

Negative-exponential demand

$$\begin{aligned} d(\mu,\pi) &= \tau e^{(1-\frac{\pi}{\mu})\lambda} \\ \epsilon(\mu,\pi) &= \frac{\lambda}{\mu}\pi \end{aligned}$$

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Multiple Equilibria

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$$f(\pi_e) = E[\text{Total Profit}] = 0$$

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$$f(\pi_e) = d(\mu_1, \pi_e)p_1(\pi_e - \mu_1) + d(\mu_2, \pi_e)p_2(\pi_e - \mu_2). \tag{1}$$

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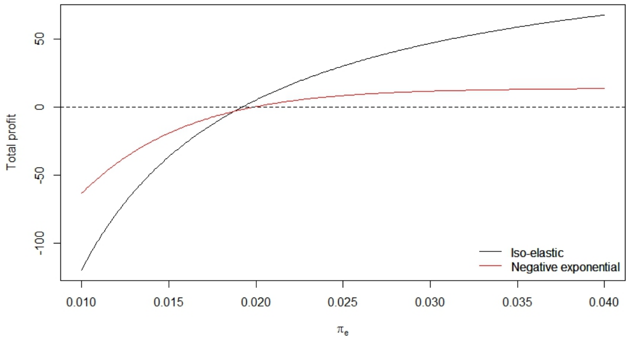
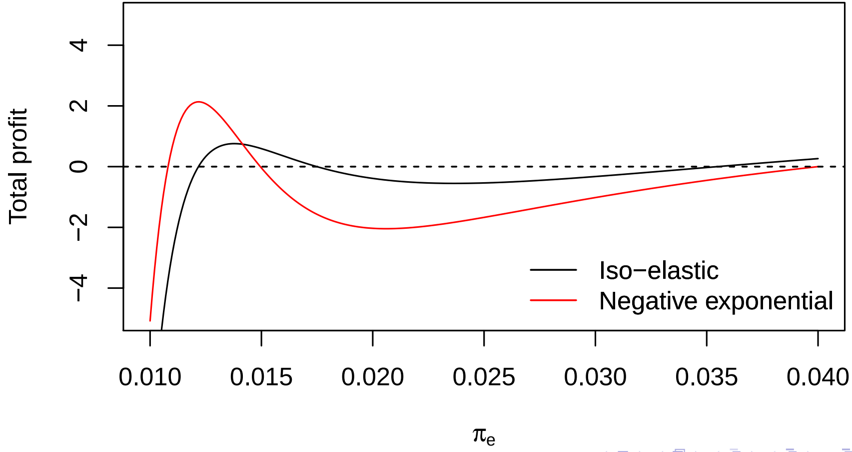


Multiple Equilibria



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- $\alpha = \frac{p_1}{p_2}$.
- π_{01}, π_{02} are solutions to $f(\pi_e) = 0, f'(\pi_e) \leq 0$.

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Theorem

Given $(\mu_1, \mu_2), (\tau_1, \tau_2)$ and (λ_1, λ_2), if $c < c_1$ and $\alpha(\pi_{01}) \leq \alpha \leq \alpha(\pi_{02})$, there are multiple equilibria. Otherwise, there is a unique equilibrium premium.

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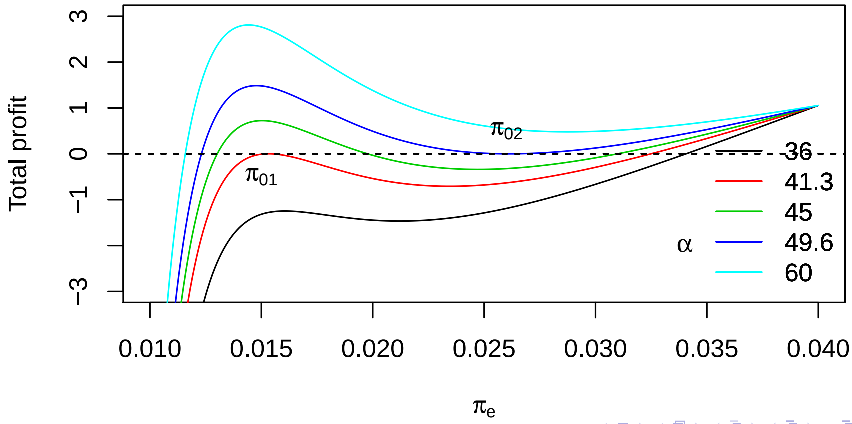
For iso-elastic demand, $c = \lambda_2 - \lambda_1, c_1 = -\frac{\sqrt{\mu_1}+\sqrt{\mu_2}}{\sqrt{\mu_2}-\sqrt{\mu_1}} < 0$.
For negative-exponential demand, $c = \frac{\lambda_2}{\mu_2} - \frac{\lambda_1}{\mu_1}, c_1 = -\frac{4}{\mu_2-\mu_1} < 0$.

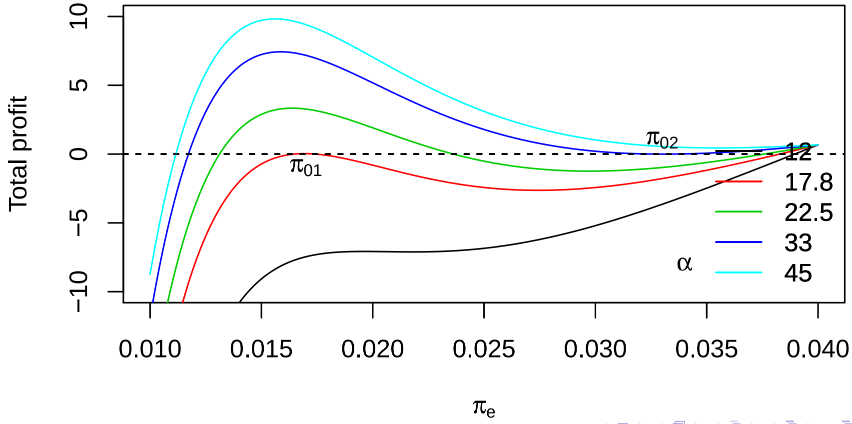
Example

Iso-elastic demand



Example

Negative-exponential demand



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Adverse Selection

- $0, \pi_1, \pi_2, \pi_3, \pi_e, \ldots, \pi_7, \pi_8, \ldots, \pi_n, 1.$

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Original definition

Purchasing decision is positively correlated with losses
-Chiappori and Salanie (2000) “Positive Correlation Test”

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- Empiriacal results are mixed and vary by market.

Life Insurance	Cawley and Philipson (1999)	X
Auto Insurance	Chiappori and Salanie (2000)	X
	Cohen (2005)	O
Annuity	Finkelstein and Poterba (2004)	X
Health Insurance	Cardon and hendel (2001)	X

Adverse Selection

- Restricting risk classification -> Policy is over-subscribed by high risks **BAD?**

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where

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Adverse Selection

- Restricting risk classification -> Policy is over-subscribed by high risks **BAD?**
- Model:

$$S = \frac{E[QL]}{E[Q]E[L]}, \quad (2)$$

where

$$Q : \text{quantity of insurance}$$
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- **A moderate degree of adverse selection can be PREFERABLE!**

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Loss Coverage

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$$\text{Loss Coverage} = \frac{\text{insured expected losses}}{\text{population expected losses}}$$

$$\text{Loss Coverage Ratio} = \frac{\text{loss coverage at a pooled premium}\pi_e}{\text{loss coverage at at fair premium}\pi_i} = \frac{\sum_{i=1}^{n} d(\mu_i, \pi_e) p_i \mu_i}{\sum_{i=1}^{n} d(\mu_i, \mu_i) p_i \mu_i}$$

Loss Coverage

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\text{Loss Coverage Ratio} &= \frac{\text{loss coverage at a pooled premium}\pi_e}{\text{loss coverage at at fair premium}\pi_i} \\
&= \frac{\sum_{i=1}^{n} d(\mu_i, \pi_e) p_i \mu_i}{\sum_{i=1}^{n} d(\mu_i, \mu_i) p_i \mu_i} > 1, \textbf{PREFERABLE!}
\end{aligned}$$

Example

Example

- A population of 1000
- Two risk groups
  - 200 high risks with risk 0.04
  - 800 low risks with risk 0.01
- No moral hazard

Example

No restriction on risk classification

Example

No restriction on risk classification

Table 1	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium (**fair premium**)	0.01	0.04
Number insured	400	100
Insured expected losses	4	4
Loss Coverage	0.5	
Loss Coverage Ratio	1	

Example

No restriction on risk classification

Table 1	Low risk-group	High risk-group
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No adverse selection.

Example

Restriction on risk classification-Case 1

Example

Restriction on risk classification-Case 1

Table 2	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium (**pooled premium** π_e)	0.02	
Number insured	300(400)	150(100)
Insured expected losses	3	6
Loss Coverage	0.5625	
Loss Coverage Ratio	**1.125>1**	

Example

Restriction on risk classification-Case 1

Table 2	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium (**pooled premium** π_e)	0.02	
Number insured	300(400)	150(100)
Insured expected losses	3	6
Loss Coverage	0.5625	
Loss Coverage Ratio	**1.125>1**	

Moderate adverse selection but preferable loss coverage.

Example

Restriction on risk classification-Case 2

Example

Restriction on risk classification-Case 2

Table 3	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium (**pooled premium** π_e)	0.02154	
Number insured	200(400)	125(100)
Insured expected losses	2	5
Loss Coverage	0.4375	
Loss Coverage Ratio	**0.875<1**	

Example

Restriction on risk classification-Case 2

Table 3	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium (**pooled premium** π_e)	0.02154	
Number insured	200(400)	125(100)
Insured expected losses	2	5
Loss Coverage	0.4375	
Loss Coverage Ratio	**0.875<1**	

Severe adverse selection and worse loss coverage.

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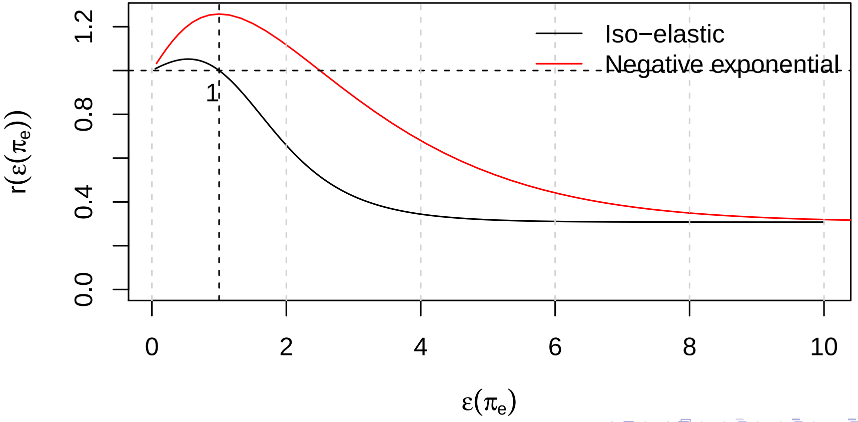
Results

Assumptions

- There are *n* risk-groups
- They have equal demand elasticities -> **Unique Equilibrium**
  For $i \neq j, i, j \in (1, n)$,
  - Iso-elastic demand: $\lambda_i = \lambda_j = \epsilon(\pi_e)$
  - Negative-exponential demand: $\frac{\lambda_i}{\mu_i}\pi_e = \frac{\lambda_j}{\mu_j}\pi_e = \epsilon(\pi_e)$

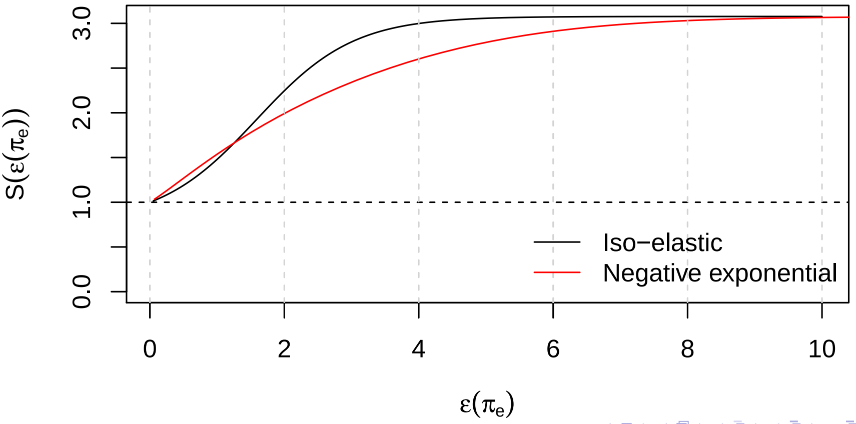
Results

Loss Coverage



Results

Adverse Selection



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  - **Adverse selection is not always a bad thing, a moderate level can increase loss coverage.**
  - This result also holds when there are *n* risk-groups with risks $\mu_1 < \mu_2 < ... < \mu_n$.
  - This result also holds when the risk $\mu \sim \mathrm{Beta}(\alpha, \beta)$.

Further Research

- Other/more general demand e.g. $d(\mu, \pi) = \tau e^{1-(\frac{\pi}{\mu})^{\lambda}}$.
- Loose restriction on demand elasticities.
- Partial restriction on risk classification.

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Questions?

Thank you!